Factors, Fractions, and Exponents

Chapter 4

Where You're Going

In this chapter, you will learn how to

- Simplify expressions with exponents.
- Simplify fractions.
- Write and calculate in scientific notation.
- Solve problems by accounting for all possibilities.

Real-World Snapshots Applying what you learn, on pages 228–229 you will solve problems about cicada emergence cycles.

LESSONS

Key Vocabulary

- base (p. 182)
- composite number (p. 186)
- divisible (p. 178)
- equivalent fractions (p. 192)
- exponents (p. 182)
- factor (p. 179)
- greatest common factor (GCF) (p. 187)
- power (p. 182)
- prime factorization (p. 187)
- prime number (p. 186)
- rational number (p. 201)
- scientific notation (p. 215)
- simplest form (p. 192)
- standard notation (p. 216)

4-1 Divisibility and Factors

What You'll Learn

- OBJECTIVE 1 To use divisibility tests
- OBJECTIVE 2 To find factors

. . . And Why

To solve real-world problems involving arrangements

✓ Check Skills You'll Need

Find each quotient.

1. 480 ÷ 3 **2.** 365 ÷ 5

3. 459 ÷ 9 **4.** 288 ÷ 6

5. $\frac{354}{2}$ **6.** $\frac{354}{3}$

For help, go to Skills Handbook, p. 760.

New Vocabulary

- divisible
- factor

Interactive lesson includes instant self-check, tutorials, and activities.

OBJECTIVE

1 Using Divisibility Tests

One integer is **divisible** by another if the remainder is 0 when you divide. Because 18 ÷ 3 = 6, 18 is divisible by 3. You can test for divisibility using mental math.

Key Concepts Divisibility Rules for 2, 5, and 10

An integer is divisible by

- 2 if it ends in 0, 2, 4, 6, or 8.
- 5 if it ends in 0 or 5.
- 10 if it ends in 0.

Even numbers end in 0, 2, 4, 6, or 8 and are divisible by 2.
Odd numbers end in 1, 3, 5, 7, or 9 and are not divisible by 2.

1 EXAMPLE Divisibility by 2, 5, and 10

Is the first number divisible by the second? Explain.

a. 567 by 2 No; 567 does not end in 0, 2, 4, 6, or 8.

b. 1,015 by 5 Yes; 1,015 ends in 5.

c. 111,120 by 10 Yes; 111,120 ends in 0.

✓ Check Understanding Example 1

1. Is the first number divisible by the second? Explain.

a. 160 by 5 **b.** 56 by 10 **c.** 53 by 2 **d.** 1,118 by 2

To see a pattern for divisibility by 3 and 9, compare the answers to the questions asked in this table.

Number	Sum of digits	Is the sum divisible by		Is the number divisible by	
		3?	9?	3?	9?
282	2 + 8 + 2 = 12	Yes	No	Yes	No
468	4 + 6 + 8 = 18	Yes	Yes	Yes	Yes
215	2 + 1 + 5 = 8	No	No	No	No
1,017	1 + 0 + 1 + 7 = 9	Yes	Yes	Yes	Yes

The pattern in the table suggests the following rules for divisibility by 3 and 9.

Key Concepts **Divisibility Rules for 3 and 9**

An integer is divisible by

- 3 if the sum of its digits is divisible by 3.
- 9 if the sum of its digits is divisible by 9.

2 EXAMPLE Divisibility by 3 and 9

Is the first number divisible by the second? Explain.

a. 567 by 3 Yes; $5 + 6 + 7 = 18$. 18 is divisible by 3.

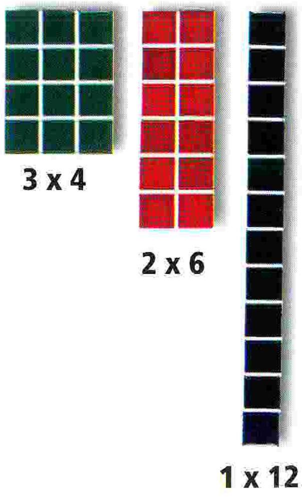
b. 1,015 by 9 No; $1 + 0 + 1 + 5 = 7$. 7 is not divisible by 9.

Check Understanding Example 2

2. Is the first number divisible by the second? Explain.

a. 64 by 9 **b.** 472 by 3 **c.** 174 by 3 **d.** 43,542 by 9

OBJECTIVE 2 Finding Factors



You can form the three rectangles at the right with 12 squares. Each rectangle has an area of 12 square units. Their dimensions, 1, 2, 3, 4, 6, and 12, are the *factors* of 12. One integer is a **factor** of another nonzero integer if it divides that integer with remainder zero.

3 EXAMPLE Real-World Problem Solving

Concerts **There are 20 students singing at a school concert. Each row of singers must have the same number of students. If there are at least 5 students in each row, what are all the possible arrangements?**

$1 \cdot 20,\ 2 \cdot 10,\ 4 \cdot 5$ **Find the factors of 20.**

There can be 1 row of 20 students, 2 rows of 10 students, or 4 rows of 5 students.

Check Understanding Example 3

3. List the positive factors of each integer.

a. 10 **b.** 21 **c.** 24 **d.** 31

e. What are the possible arrangements for Example 3 if there are 36 students singing at the concert?

EXERCISES

For more exercises, see *Extra Practice*.

Practice and Problem Solving

A Practice by Example

Example 1 (page 178)

Is the first number divisible by the second? Explain.

1. 20 by 10
2. 37 by 2
3. 45 by 5
4. 240 by 2
5. 60 by 5
6. 123 by 2
7. 1,468 by 2
8. 2,005 by 10

Example 2 (page 179)

9. 78 by 9
10. 69 by 3
11. 108 by 9
12. 258 by 3
13. 3,694 by 9
14. 5,751 by 9
15. 123 by 3
16. 456 by 3

Example 3 (page 179)

List the positive factors of each integer.

17. 4
18. 8
19. 23
20. 75

21. Drill Team There are 32 students in the school drill team performance. Each row of team members must have the same number of students. If there are at least 8 students in each row, what are all the possible arrangements?

B Apply Your Skills

State whether each number is divisible by 2, 3, 5, 9, 10, or none. Explain. Some numbers may have more than one divisor.

22. 111
23. 131
24. 288
25. 300
26. 52
27. 891
28. 4,805
29. 437,684

30. a. Which of the following numbers are divisible by both 2 and 3?
10 66 898 4,710 975
b. Which of the numbers above are divisible by 6?
c. Using your results, write a divisibility rule for 6.

Show all possible ways that each integer can be written as the product of two positive factors.

31. 25
32. 28
33. 32
34. 35
35. 37
36. 50
37. 53
38. 72

Write the missing digit to make each number divisible by 9.

39. 22■,034
40. 3■,817
41. 2,03■,371
42. 1■,111

43. Writing in Math If a number is divisible by 9, is it also divisible by 3? Explain how you reached your conclusion.

44. Reasoning John made oatmeal cookies for a class bake sale. The cookies need to be distributed equally on 2 or more plates. If each plate gets at least 7 cookies, what are the possible combinations for the totals below?
a. 42 cookies
b. 56 cookies
c. 60 cookies
d. 144 cookies

Challenge

45. **a.** Copy and complete the table.

Number	Last two digits	Are last two digits divisible by 4?	Is the number divisible by 4?
136	36	Yes	Yes
1,268	68	Yes	Yes
314	14	No	No
1,078	■	■	■
696	■	■	■

b. **Reasoning** Write a divisibility rule for 4.

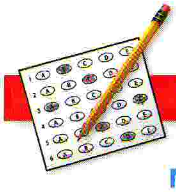
Open-Ended **Write three numbers greater than 20 that match each description.**

46. Divisible by 5, but not divisible by 10

47. Divisible by 3, but not divisible by 5, 9, or 10

48. Divisible by 2, 3, 5, and 10, but not divisible by 9

49. **Reasoning** If a is divisible by 2, what can you conclude about $a + 1$? Justify your answer.



Test Prep

Multiple Choice

50. Which list shows all the positive factors of 15?
A. 1, 15 **B.** 1, 3, 15 **C.** 1, 5, 15 **D.** 1, 3, 5, 15

51. Which list shows all the positive factors of 17?
F. 1, 17 **G.** 1, 7, 17 **H.** 1, 2, 7, 17 **I.** 1, 2, 8, 17

52. Which number is NOT a factor of 438?
A. 2 **B.** 3 **C.** 5 **D.** 6

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Short Response

53. **a.** What three positive numbers less than 100 are divisible by 2, 3, and 5?
b. Justify your answer.

Mixed Review

Lesson 3-7 **Complete each statement.**

54. 24 ■ = 24,000 mg

55. 18.2 km = 1,820,000 ■

Lesson 2-9

56. **Grocery Shopping** You have \$5 to spend at the grocery store. You need \$2.89 for a gallon of milk. Write and solve an inequality to show how much money m you can spend on a box of cereal.

Lesson 1-3 **Evaluate.**

57. $3y + 3$, for $y = 8$

58. $4(2 + a)$, for $a = 10$

4-2 Exponents

What You'll Learn

OBJECTIVE 1 To use exponents

OBJECTIVE 2 To use the order of operations with exponents

. . . And Why

To solve real-world problems involving magnification

✓ Check Skills You'll Need

Find each product.

1. $3 \cdot 3 \cdot 3 \cdot 3$
2. $-12 \cdot (-12)$
3. $(-4)(-4)(-4)$
4. $10 \cdot 10 \cdot 10 \cdot 10$

For help, go to Lesson 1-9.

New Vocabulary

- exponents
- power
- base

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OBJECTIVE

1 Using Exponents

You can use **exponents** to show repeated multiplication.

$$\text{base} \rightarrow \overset{\text{exponent}}{\underset{\text{power}}{\underbrace{2^6}}} = \underbrace{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2}_{\text{The base 2 is used as a factor 6 times.}} = 64 \leftarrow \text{the value of the expression}$$

A **power** has two parts, a **base** and an exponent. The expression 2^6 is read as "two to the sixth power."

Power	Verbal Expression	Value
12^1	*Twelve to the first power*	12
6^2	*Six to the second power,* or *six squared*	$6 \cdot 6 = 36$
$(0.2)^3$	*Two tenths to the third power,* or *two tenths cubed*	$(0.2)(0.2)(0.2) = 0.008$
-7^4	*The opposite of the quantity seven to the fourth power*	$-(7 \cdot 7 \cdot 7 \cdot 7) = -2{,}401$
$(-8)^5$	*Negative eight to the fifth power*	$(-8)(-8)(-8)(-8)(-8) = -32{,}768$

1 EXAMPLE Using an Exponent

Write the expression using an exponent.

a. $(-5)(-5)(-5)$

$(-5)^3$ Include the negative sign within parentheses.

b. $-2 \cdot a \cdot b \cdot a \cdot a$

$-2 \cdot a \cdot a \cdot a \cdot b$ Rewrite the expression using the Commutative and Associative Properties.

$-2a^3b$ Write $a \cdot a \cdot a$ using exponents.

✓ Check Understanding Example 1

1. Write using exponents.

a. $6 \cdot 6 \cdot 6$ **b.** $4 \cdot y \cdot x \cdot y$ **c.** $(-3)(-3)(-3)(-3)$

2 EXAMPLE Real-World Problem Solving

Science A microscope can magnify a specimen 10^3 times. How many times is that?

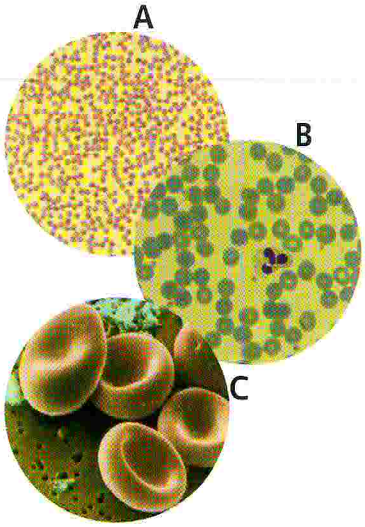
$10^3 = 10 \cdot 10 \cdot 10$ **The exponent indicates that the base 10 is used as a factor 3 times.**

$= 1{,}000$ **Multiply.**

The microscope can magnify the specimen 1,000 times.

Check Understanding Example 2

2. a. Simplify 7^2. b. Evaluate $-a^4$ and $(-a)^4$, for $a = 2$.



Real-World Connection

Human blood cells are shown here magnified (A) 10^2 times, (B) 10^3 times, and (C) 10^4 times.

OBJECTIVE

2 Using the Order of Operations With Exponents

You can extend the order of operations to include exponents.

Key Concepts Order of Operations

1. Work inside grouping symbols.
2. Simplify any terms with exponents.
3. Multiply and divide in order from left to right.
4. Add and subtract in order from left to right.

3 EXAMPLE Using the Order of Operations

a. Simplify $4(3 + 2)^2$.

$4(3 + 2)^2 = 4(5)^2$ **Work within parentheses first.**

$= 4 \cdot 25$ **Simplify 5^2.**

$= 100$ **Multiply.**

b. Evaluate $-2x^3 + 4y$, for $x = -2$ and $y = 3$.

$-2x^3 + 4y = -2(-2)^3 + 4(3)$ **Replace x with -2 and y with 3.**

$= -2(-8) + 4(3)$ **Simplify $(-2)^3$.**

$= 16 + 12$ **Multiply from left to right.**

$= 28$ **Add.**

Reading Math

The expression $4(3 + 2)^2$ is read as "four times the square of the quantity three plus two."

Check Understanding Example 3

3. a. Simplify $2 \cdot 5^2 + 4 \cdot (-3)^3$.
   b. Evaluate $3a^2 + 6$, for $a = -5$.

EXERCISES

For more exercises, see *Extra Practice.*

Practice and Problem Solving

A Practice by Example

Example 1 (page 182)

Write using exponents.

1. $8 \cdot 8 \cdot 8$
2. $r \cdot r \cdot r \cdot r \cdot s \cdot s$
3. $-7 \cdot a \cdot a \cdot b$
4. $5 \cdot 5 \cdot a \cdot a$
5. $9 \cdot 9 \cdot 9 \cdot 9 \cdot 9$
6. $(-5)(-5)(-5)(-5)$

Example 2 (page 183)

Simplify.

7. 10^4
8. 4^3
9. 2^6
10. $(-4)^3$
11. -4^3
12. $(-6)^3$

13. **Science** An electron microscope can magnify a specimen about 10^6 times. How many times is that?

Example 3 (page 183)

Simplify.

14. $3(4 + 2)^2$
15. $49 - (4 \cdot 2)^2$
16. $-3^2 + 5 \cdot 2^3$
17. $2(9 - 4)^2$
18. $25 - (3 \cdot 2)^2$
19. $2 \cdot (-2)^4 + 10^1$

Evaluate.

20. $3a^2 - 2$, for $a = 5$
21. $c^3 + 4$, for $c = -6$
22. $-4y^2 + y^3$, for $y = 3$
23. $2m^2 + n$, for $m = -3$ and $n = 4$

B Apply Your Skills

Write using exponents.

24. $-5 \cdot x \cdot x \cdot 3 \cdot y$
25. d cubed
26. $-2 \cdot a \cdot (-4) \cdot b \cdot b$

27. **Error Analysis** A student gives ab^3 as an answer when asked to write the expression $ab \cdot ab \cdot ab$ using exponents. What is the student's error?

Simplify.

28. -1^8 and $(-1)^8$
29. -2^4 and $(-2)^4$
30. $15 + (4 + 6)^2 \div 5$
31. $(-4)(-6)^2 (2)$
32. $(4 + 8)^2 \div 4^2$
33. $(12 - 3)^2 \div (2^2 - 1^2)$

Evaluate each expression.

34. $-6m^2$, for $m = 2$
35. $5k^2$, for $k = 1.2$
36. $8 - x^3$, for $x = -2$
37. $3(2m + 5)^2$, for $m = 2$
38. $4(2y - 3)^2$, for $y = 5$
39. $y^2 + 2y + 5$, for $y = -6$

n	$4n$	4^n	n^4
1	■	■	■
2	■	■	■
3	■	■	■
4	■	■	■

40. a. Copy and complete the table at the left.
    b. For what value(s) of n is each sentence true?
    $4^n = n^4$ $\quad$ $4^n < n^4$ $\quad$ $4^n > n^4$

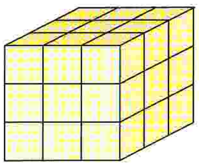
41. **Reasoning** Does $-a^2 = (-a)^2$ for any values of a? Explain.

42. **Mental Math** Given that $2^{10} = 1{,}024$, find 2^{11} mentally.

43. Read the word phrase that follows:
the square of *a*, increased by the sum of twice *a* and 3.
a. Write a variable expression for the word phrase.
b. Evaluate the expression for $a = 7$.

Challenge

Geometry **Exercises 44–48 involve cubes made from smaller cubes, like the one at the left. Suppose such a cube has edges of length 5 cm.**



Edge length $s = 3$ in.
Area of Face $= s^2$
$= 9$ in.^{2}
Volume $= s^3$
$= 27$ in.^{3}

44. What is the area of a face?

45. What is the volume?

What is the length of an edge of a cube that has the following measurement?

46. face area of 64 in.^{2}

47. a volume of 64 in.^{3}

48. **Language Arts** Why do you think *squared* and *cubed* are used to indicate the second power and the third power?

49. **Reasoning** Describe all pairs of values of *x* and *y* for which $5x^2y = 5xy^2$. Justify your answer.

50. **Writing in Math** Evaluate $(-1)^m$ for $m = 2, 4,$ and 6. Then evaluate $(-1)^m$ for $m = 1, 3,$ and 5. Write a conjecture about the sign of an even power of a negative number. Then write a conjecture about the sign of an odd power of a negative number.

Test Prep

Multiple Choice

51. What is the value of $(0.5)^2$?
A. 0.1 **B.** 0.25 **C.** 1.0 **D.** 25

52. What is the value of xy^2 for $x = 3$ and $y = 4$?
F. 12 **G.** 24 **H.** 36 **I.** 48

53. Which expression equals 1?
A. -1^2 **B.** $(-1)^3$ **C.** $-(-1)^2$ **D.** $|-1|^3$

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Short Response

54. Is $a^3 \geq a$ for all integer values of *a*? Explain and give an example.

Mixed Review

Lesson 4-1 **State whether each number is divisible by 2, 3, 5, 9, 10, or none.**

55. 36 **56.** 135 **57.** 171 **58.** 190
59. 253 **60.** 123 **61.** 117 **62.** 30

Lesson 3-3

63. **a.**Sara's grades are 79, 82, 75, 86, and 93. What is the mean?
b. What is the median?

Lesson 2-3 **Simplify each expression.**

64. $3x - 2y + x$ **65.** $w + 8 - 4w - 15$ **66.** $9a + 2(a - 5) + 3$

4-3 Prime Factorization and Greatest Common Factor

What You'll Learn

OBJECTIVE 1 To find the prime factorization of a number

OBJECTIVE 2 To find the greatest common factor (GCF) of two or more numbers

. . . And Why

To solve real-world problems involving organization

Check Skills You'll Need

List the positive factors of each number.

1. 15 **2.** 35 **3.**7

4. 20 **5.** 100 **6.**121

For help, go to Lesson 4-1.

New Vocabulary

- prime number
- composite number
- prime factorization
- greatest common factor (GCF)

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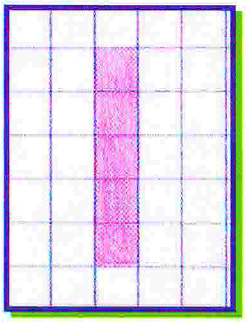
OBJECTIVE 1 Finding Prime Factorizations

Investigation
Exploring Prime Numbers

The diagram shows the only rectangle you can make with integer side lengths and an area of 5 square units. Work with a partner. Find the number of rectangles you can make with each number of unit squares: 2, 3, 4, 5, 6, 7, 8, 9, and 10.



1. For which numbers of squares is only one rectangle possible?
2. For which numbers of squares is more than one rectangle possible?
3. List the dimensions of the rectangles you can make with each of the following numbers of unit squares: 13, 15, 17, 19, and 21.

A **prime number** is an integer greater than 1 with exactly two positive factors, 1 and the number itself. The numbers 2, 3, 5, and 7 are prime numbers.

A **composite number** is an integer greater than 1 with more than two positive factors. The numbers 4, 6, 8, 9, and 10 are composite numbers. The number 1 is neither prime nor composite.

1 EXAMPLE Prime or Composite?

State whether each number is *prime* or *composite*. Explain.

a. 23 Prime; it has only two factors, 1 and 23.

b. 129 Composite; it has more than two factors, 1, 3, 43, and 129.

Check Understanding Example 1

1. Which numbers from 10 to 20 are prime? Which are composite?

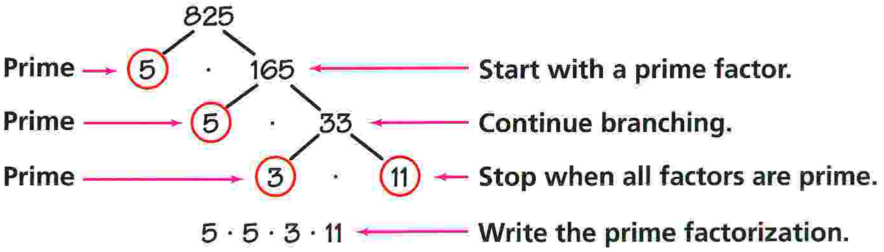
Writing a composite number as a product of its prime factors shows the **prime factorization** of the number. You can use a *factor tree* to find prime factorizations. Write the final factors in increasing order from left to right. Use exponents to indicate repeated factors.

Test-Taking Tip

To check whether a number is prime, look for prime factors in order, starting with 2. When you get to a prime whose square is greater than the original number, you can stop. For 23, check 2 and 3. Then stop at 5, since $5^2 > 23$. Since 2, 3, and 5 are not factors of 23, 23 is prime.

2 EXAMPLE Writing the Prime Factorization

Use a factor tree to write the prime factorization of 825.



Start with a prime factor.

Continue branching.

Stop when all factors are prime.

Write the prime factorization.

$825 = 3 \cdot 5^2 \cdot 11$ **Use exponents to write the prime factorization.**

Check Understanding Example 2

2. Write the prime factorization of each number.

a. 72 **b.** 121 **c.** 225 **d.** 236

OBJECTIVE 2 Finding the Greatest Common Factor

Factors that are the same for two or more numbers or expressions are *common factors*. The greatest of these common factors is called the **greatest common factor (GCF).** You can use prime factorization to find the GCF of two or more numbers or expressions. If there are no prime factors and variable factors in common, the GCF is 1.

3 EXAMPLE Finding the GCF

Find the GCF of each pair of numbers or expressions.

a. 40 and 60

$$40 = \boxed{2^3} \cdot \boxed{5}$$
$$60 = \boxed{2^2} \cdot 3 \cdot \boxed{5}$$

$$\text{GCF} = 2^2 \cdot 5 = 20$$

The GCF of 40 and 60 is 20.

Write the prime factorizations.

Find the common factors. Use the lesser power of the common factors.

b. $6a^3b$ and $4a^2b$

$$6a^3b = \boxed{2} \cdot 3 \cdot \boxed{a^3} \cdot \boxed{b}$$
$$4a^2b = \boxed{2} \cdot 2 \cdot \boxed{a^2} \cdot \boxed{b}$$

$$\text{GCF} = 2 \cdot a^2 \cdot b = 2a^2b$$

The GCF of $6a^3b$ and $4a^2b$ is $2a^2b$.

Check Understanding **Example 3**

3. Use prime factorizations to find each GCF.

a. 8, 20 **b.** 12, 87 **c.** $12r^3, 8r$ **d.** $15m^2n, 45m$

You can find the GCF of two or more numbers or expressions by listing factors or by using prime factorizations.

More Than One Way

A parade organizer wants each of three marching bands to have the same number of band members in each row. The bands have 48, 32, and 56 band members. What is the greatest number of band members possible for each row?

Jasmine's Method

List the factors of each number. Then find the greatest factor the numbers have in common.

48: 1, 2, 3, 4, 6, 8, 12, 16, 24, 48
32: 1, 2, 4, 8, 16, 32
56: 1, 2, 4, 7, 8, 14, 28, 56

The GCF of 48, 32, and 56 is 8. The greatest possible number of band members in each row is 8.

Daryl's Method

Find the prime factorization of each number. Then find the least power of all common prime factors.

48: $2^4 \cdot 3$
32: 2^5
56: $2^3 \cdot 7$

The GCF of 48, 32, and 56 is 2^3, or 8. The greatest possible number of band members in each row is 8.

Choose a Method

1. Which method do you prefer to find the GCF? Explain why.
2. Which method would you use to find the GCF of 4, 8, and 24? Of 54, 27, and 36? Explain why.

EXERCISES

For more exercises, see *Extra Practice.*

Practice and Problem Solving

A Practice by Example

Example 1 (page 186)

State whether each number is prime or composite. Explain.

1. 27
2. 19
3. 31
4. 38
5. 45
6. 53
7. 87
8. 93

Example 2 (page 187)

Write the prime factorization of each number.

9. 8
10. 49
11. 34
12. 42
13. 360
14. 115
15. 186
16. 621

Example 3 (page 187)

Use prime factorization to find each GCF.

17. 10, 45
18. 14, 21
19. 25, 100
20. 57, 84
21. $14c^2, 35c$
22. $3y^2, 24y^3$
23. $18c^3, 24c^3$
24. $6m^3n, 8mn^2$

B Apply Your Skills

Is each number prime, composite, or neither? For each composite number, write the prime factorization.

25. 17
26. 1
27. 49
28. 522

29. Organization A math teacher and a science teacher combine their first-period classes for a group activity. The math class has 24 students and the science class has 16 students. The teachers need to divide the students into groups of the same size. Each group must have the same number of math students. Find the greatest number of groups possible.

Find each GCF.

30. 6, 8, 12
31. 42, 65
32. 54, 144
33. 8, 16, 20
34. 12, 18, 21
35. 143, 169
36. z, z^2
37. $180a^2, 210a$
38. x^2y, xy^2
39. a^3b, a^2b^2
40. c^3df^2, c^2d^2f
41. a^2b, b^2c, ac^2

42. Reasoning Find the integers that fit the following conditions:
- They are between 44 and 53.
- The sums of their digits are prime.
- They have more than three factors.

43. Open-Ended The GCF of 36 and x is 6. What are two possible values for x?

44. Seating Arrangements Organizers for a high school graduation have set up chairs in two sections. They put 126 chairs for graduates in the front section and 588 chairs for guests in the back section. If all rows have the same number of chairs, what is the greatest number of chairs possible for a row?

C Challenge

Is each number prime, composite, or neither? For each composite number, write the prime factorization.

45. 253 **46.** 1,575 **47.** 1,003 **48.** 283

Two numbers are *relatively prime* if their GCF is 1. Are the numbers in each pair below relatively prime? Explain.

SAMPLE 8, 17 Yes, 8 and 17 are relatively prime. The GCF is 1.
7, 35 No, 7 and 35 are not relatively prime. The GCF is 7.

49. 3, 20 **50.** 9, 42 **51.** 13, 52 **52.** 24, 47

53. 52, 65 **54.** 63, 74 **55.** 15, 22 **56.** 42, 72

57. **Writing in Math** Explain how to find the prime factorization of 50.

Test Prep

Multiple Choice

In Exercises 58 and 59, what is the GCF of each given pair?

58. $27x^2y^3$ and $46x^2y$
A. $3x^2y$ **B.** x^2y^2 **C.** x^2y **D.** $9x^2y$

59. $25b^2c$ and $42bc$
F. bc^2 **G.** bc **H.** b^2c **I.** b^2c^2

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60. For which pair is the GCF 12?
A. 3, 4 **B.** $24x^2$, $36y$ **C.** $12xy$, $24y$ **D.** $3x$, $12x$

61. Simon is covering a wall with equal-sized tiles that cannot be cut into smaller pieces. The wall is 66 inches high by 72 inches wide. What is the area of the largest square tile that Simon can use?
F. 9 in.^{2} **G.** 16 in.^{2} **H.** 36 in.^{2} **I.** 64 in.^{2}

Extended Response

62. **a.** Is the product of two prime numbers also prime?
b. Justify your answer. **c.** Give an example.

Mixed Review

Lesson 4-2 **Evaluate for $x = 2$ and $y = 5$.**

63. x^2y **64.** xy^2 **65.** $x^2 + y^2$ **66.** $x^4 - y$

Lesson 3-6 **Solve each equation.**

67. $3x = 5.4$ **68.** $-0.5a = 4.35$

69. $4.32 = 1.6y$ **70.** $-8m = -74.4$

Lesson 2-7 **71.** **Bookstore** A store manager ordered three times as many books as magazines. She ordered a total of 108 books and magazines. How many books did she order?

Extension

Venn Diagrams

For Use With Lesson 4-3

In a *Venn diagram* you use circles to represent collections of objects. The *intersection,* or overlap, of two circles indicates what is common to both collections.

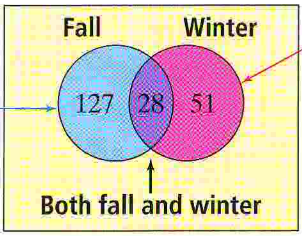
1 EXAMPLE

School coaches plan to send notices to all students playing fall or winter sports. How many notices do they need to send?

Students in Sports

Season	Students
Fall	155
Winter	79
Both fall and winter	28

number who played only a fall sport 155 − 28 = 127



number who played only a winter sport 79 − 28 = 51

Add all three numbers to find the number of notices needed.

127 + 28 + 51 = 206

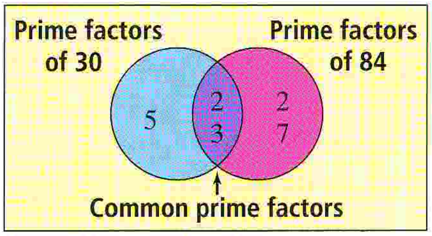
The coaches need to send 206 notices.

You can use a Venn diagram to find the GCF of two numbers.

2 EXAMPLE

Find the GCF of 30 and 84.

Include the common prime factors of 30 and 84 in the intersection.



The GCF is the product of the factors in the intersection.

The GCF is 2 · 3, or 6.

EXERCISES

1. In a class of 38 students, 32 are wearing jeans, 21 are wearing T-shirts, and 15 are wearing both. How many students are wearing jeans and something other than a T-shirt?

Draw a Venn diagram to find the GCF of each pair of numbers.

2. 24, 56 **3.** 35, 49 **4.** 36, 84 **5.** 72, 108

4-4 Simplifying Fractions

What You'll Learn

OBJECTIVE 1 To find equivalent fractions

OBJECTIVE 2 To write fractions in simplest form

. . . And Why

To solve real-world problems involving statistics

Check Skills You'll Need

Find each GCF.

1. 14, 21 **2.** 48, 60

3. $5mn, 15m^2n$

4. $63r^2, 48s^3$

For help, go to Lesson 4-3.

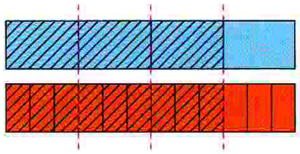
New Vocabulary

- **equivalent fractions**
- **simplest form**

OBJECTIVE

1 Finding Equivalent Fractions

Each fraction model below represents one whole. The blue model is divided into four equal parts. The orange model is divided into twelve equal parts.



$\frac{3}{4}$ of the model is shaded.

$\frac{9}{12}$ of the model is shaded.

$$\frac{3}{4} = \frac{3 \cdot 3}{4 \cdot 3} = \frac{9}{12}$$

The fraction models show that $\frac{3}{4} = \frac{9}{12}$. The fractions $\frac{3}{4}$ and $\frac{9}{12}$ are **equivalent fractions** because they describe the same part of a whole.

You can find equivalent fractions by multiplying or dividing the numerator and denominator by the same nonzero factor.

1 EXAMPLE Finding an Equivalent Fraction

Find two fractions equivalent to $\frac{4}{12}$.

a. $\frac{4}{12} = \frac{4 \cdot 3}{12 \cdot 3}$

$= \frac{12}{36}$

b. $\frac{4}{12} = \frac{4 \div 4}{12 \div 4}$

$= \frac{1}{3}$

The fractions $\frac{12}{36}$ and $\frac{1}{3}$ are both equivalent to $\frac{4}{12}$.

Check Understanding Example 1

1. Find two fractions equivalent to each fraction.

a. $\frac{5}{15}$ **b.** $\frac{10}{12}$ **c.** $\frac{14}{20}$

Reading Math

Most fraction names are made by adding *th* or *ths* to the denominator. You read $\frac{1}{4}$ as "one fourth," $\frac{2}{5}$ as "two fifths," and $\frac{8}{10}$ as "eight tenths." Halves and thirds are two exceptions.

OBJECTIVE

2 Writing Fractions in Simplest Form

A fraction is in **simplest form** when the numerator and the denominator have no common factors other than 1. You can use the GCF to write a fraction in simplest form.

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2 EXAMPLE Real-World Problem Solving

Statistics You survey your friends about their favorite sandwich and find that 8 out of 12, or $\frac{8}{12}$, prefer peanut butter. Write this fraction in simplest form.

The GCF of 8 and 12 is 4.

$$\frac{8}{12} = \frac{8 \div 4}{12 \div 4}$$ **Divide the numerator and denominator by the GCF, 4.**

$$= \frac{2}{3}$$ **Simplify.**

The favorite sandwich of $\frac{2}{3}$ of your friends is peanut butter.

Real-World Connection

The average American child will eat 1,500 peanut butter and jelly sandwiches by the time she or he graduates from high school.

✓ Check Understanding Example 2

2. Write each fraction in simplest form.

a. $\frac{6}{8}$ **b.** $\frac{9}{12}$ **c.** $\frac{28}{35}$

You can often simplify a fraction that contains a variable. In this book, you may assume that no expression for a denominator equals zero.

3 EXAMPLE Simplifying a Fraction

Write in simplest form.

a. $\frac{y}{xy}$

$$\frac{y}{xy} = \frac{\cancel{y}^1}{x\cancel{y}_1}$$ **Divide the numerator and denominator by the common factor, *y*.**

$$= \frac{1}{x}$$ **Simplify.**

b. $\frac{3ab^2}{12ac}$

$$\frac{3ab^2}{12ac} = \frac{3 \cdot a \cdot b \cdot b}{2 \cdot 2 \cdot 3 \cdot a \cdot c}$$ **Write as a product of prime factors.**

$$= \frac{\cancel{3}^1 \cdot \cancel{a}^1 \cdot b \cdot b}{2 \cdot 2 \cdot {}_1\cancel{3} \cdot {}_1\cancel{a} \cdot c}$$ **Divide the numerator and denominator by the common factors.**

$$= \frac{b \cdot b}{2 \cdot 2 \cdot c}$$ **Simplify.**

$$= \frac{b \cdot b}{4 \cdot c}$$

$$= \frac{b^2}{4c}$$

Test-Taking Tip

You will see the directions *write in lowest terms* on some tests. This is another way of saying "write in simplest form."

✓ Check Understanding Example 3

3. Write in simplest form.

a. $\frac{b}{abc}$ **b.** $\frac{2mn}{6m}$ **c.** $\frac{24x^2y}{8xy}$

EXERCISES

For more exercises, see *Extra Practice.*

Practice and Problem Solving

A Practice by Example

Example 1 (page 192)

Find two fractions equivalent to each fraction.

1. $\frac{2}{8}$ **2.** $\frac{8}{10}$ **3.** $\frac{3}{9}$ **4.** $\frac{8}{36}$ **5.** $\frac{6}{18}$ **6.** $\frac{20}{22}$

Example 2 (page 193)

Write each fraction in simplest form.

7. $\frac{3}{9}$ **8.** $\frac{4}{10}$ **9.** $\frac{12}{48}$ **10.** $\frac{2}{10}$ **11.** $\frac{4}{12}$ **12.** $\frac{6}{15}$

13. Health Doctors suggest that most people need about 8 h of sleep each night to stay healthy. What fraction of the day is this? Write your answer in simplest form.

Example 3 (page 193)

Write in simplest form.

14. $\frac{2x}{3x}$ **15.** $\frac{4km^2}{12k}$ **16.** $\frac{b}{bc}$ **17.** $\frac{24x}{16}$

18. $\frac{8pr}{12p}$ **19.** $\frac{14a^2}{24a}$ **20.** $\frac{4bc}{16b}$ **21.** $\frac{40ab^2}{5ab}$

B Apply Your Skills

Find two fractions equivalent to each fraction.

22. $\frac{4}{8}$ **23.** $\frac{4}{10}$ **24.** $\frac{5}{20}$ **25.** $\frac{10}{16}$ **26.** $\frac{18}{20}$ **27.** $\frac{25}{100}$

Write in simplest form.

28. $\frac{8}{14}$ **29.** $\frac{18}{32}$ **30.** $\frac{20}{30}$ **31.** $\frac{12}{16}$

32. $\frac{15^3}{15^2}$ **33.** $\frac{56pq}{7pq}$ **34.** $\frac{5c^2d}{15c}$ **35.** $\frac{4r^3st}{36st^2}$

36. $\frac{5t}{10t^2}$ **37.** $\frac{x^2y}{3yz}$ **38.** $\frac{12gh}{8g^2h^2}$ **39.** $\frac{6m^2n^2}{9mn^2}$

Reading Math

For help with reading and solving Exercise 40, see page 196.

40. Error Analysis A student claims $\frac{65}{91}$ is in simplest form. Do you agree? Explain.

41. Open-Ended Write two fractions whose simplest form is $\frac{3x}{5}$.

42. Writing in Math Does $\frac{1}{2}$ of one pizza represent the same amount as $\frac{1}{2}$ of another pizza? Justify your answer.

C Challenge

Data Analysis The table shows the number of personal computers (PCs) and households with Internet access in the United States. For Exercises 43–45, write each fraction in simplest form.

PC and On-Line Households in the U.S. (millions)

Households	1997	1998
Total households	100	101
Households with PCs	44	48
Households with Internet access	21	27

SOURCE: *The Wall Street Journal Almanac 1999*

43. In 1997, what fraction of U.S. households had PCs?

44. In 1997, what fraction of U.S. households with PCs had Internet access? (Assume that a household with Internet access had a PC.)

45. a. In 1998, what fraction of U.S. households with PCs had Internet access? (*Hint:* See Exercise 44.)

b. Was the fraction greater in 1997 or 1998? Explain.

46. Write the numerator and denominator of $\frac{24}{32}$ as products of prime factors. Then use the prime factors to write $\frac{24}{32}$ in simplest form.

Test Prep

Multiple Choice

47. Which fraction is equivalent to $\frac{15}{30}$?

A. $\frac{2}{4}$ **B.** $\frac{3}{5}$ **C.** $\frac{3}{4}$ **D.** $\frac{5}{6}$

48. What is the simplest form for $\frac{14}{42}$?

F. $\frac{1}{3}$ **G.** $\frac{7}{21}$ **H.** $\frac{2}{6}$ **I.** $\frac{2}{3}$

49. What is the simplest form for $\frac{6m}{15m}$?

A. $\frac{2m}{15m}$ **B.** $\frac{3m}{5m}$ **C.** $\frac{3}{5}$ **D.** $\frac{2}{5}$

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Short Response

50. **a.** Is $\frac{ab}{5}$ equivalent to $\frac{15a^2b}{75a}$? **b.** Justify your answer.

Mixed Review

Lesson 4-3

Find the GCF for each pair.

51. 10, 12 **52.** 28, 60 **53.** $14a, 21a$ **54.** $24x^2, 40x^3$

Lesson 3-5

Solve each equation.

55. $y + 3.23 = 5.85$ **56.** $b - 2.13 = 9.9$ **57.** $12.8 + z = 6.47$

Lesson 3-2

58. **Oil Spills** A damaged oil tanker spilled 34.7 million gallons of crude oil over 4 days. On the average, about how many gallons did the tanker spill each day?

Checkpoint Quiz 1

Lessons 4-1 through 4-4

Instant self-check quiz online and on CD-ROM

State whether each number is divisible by 2, 3, 5, 9, 10, or none.

1. 30 **2.** 54 **3.** 48 **4.** 161 **5.** 2,583

Evaluate each expression.

6. x^2, for $x = 8$ **7.** a^3, for $a = 5$ **8.** $-2z^2$, for $z = -3$

Write in simplest form.

9. $\frac{8}{16}$ **10.** $\frac{14}{21}$ **11.** $\frac{16}{28}$ **12.** $\frac{3a}{12a}$ **13.** $\frac{2xy}{x}$

14. **Open-Ended** Write two expressions whose GCF is $5a^2$.

Reading Math

Reading for Problem Solving

For Use With Page 194, Exercise 40

Read the exercise below and then follow along with what Tina thinks and writes. Check your understanding by solving the exercises at the bottom of the page.

Error Analysis A student claims $\frac{65}{91}$ is in simplest form. Do you agree? Explain.

What Tina Thinks and Writes

Do I agree?	The wording of the problem suggests that the student is wrong. I'll write: No. Now I have to "Explain."
What does simplest form mean for a fraction?	The numerator and denominator can have no common factor other than 1.
65 ends in 5, so 65 has 5 as a factor. Is 5 also a factor of 91?	No! 91 does not end in 5. 5 is not a factor of 91.
Are there other possibilities for common factors?	Since 5 is a factor of 65. There has to be another factor. 65 = 5 · 13
Is 13 a factor of 91?	91 = 7 · 13. A ha! I'll finish: No. $\frac{65}{91} = \frac{5 \cdot \overset{1}{\cancel{13}}}{7 \cdot \underset{1}{\cancel{13}}} = \frac{5}{7}$ $\frac{5}{7}$ is the simplest form for the fraction. I'm done!

EXERCISES

Use what you know about factors to decide whether each fraction is in simplest form. If not, simplify.

1. $\frac{17}{51}$ **2.** $\frac{39}{91}$ **3.** $\frac{51}{57}$ **4.** $\frac{57}{76}$ **5.** $\frac{57}{87}$

Account for All Possibilities

Problem Solving 4-5

OBJECTIVE

1 Account for All Possibilities

Math Strategies in Action Have you ever lost something that you just couldn't find anywhere? Don't you usually discover that you didn't check *every* place you could, even when you thought you had?

"Has anyone seen the remote?"

Even for a situation like losing a TV remote control, making a list of places to search might help.

In some problems, you need to count the possibilities. To solve these problems, you need to be sure that you have found every possibility. Organized lists and diagrams help you keep track of the possibilities as you find them.

What You'll Learn

OBJECTIVE 1 To find all possibilities when you solve a problem

. . . And Why

To solve real-world problems involving photography

✓ Check Skills You'll Need

Compare.
Use > or < to complete each statement.

1. 3 ▨ 0

2. −16 ▨ −25

3. 0 ▨ 1

4. −30 ▨ −20

For help, go to Skills Handbook, p. 775.

1 EXAMPLE Real-World Problem Solving

Photography Mandy, Jim, Keisha, Darren, Lin, Chris, and Jen are friends. They want to take pictures of themselves with two people in each picture. How many pictures do they need to take?

Read and Understand

1. What do you need to find?
2. How many people are there in all?
3. How many people will be in each photograph?

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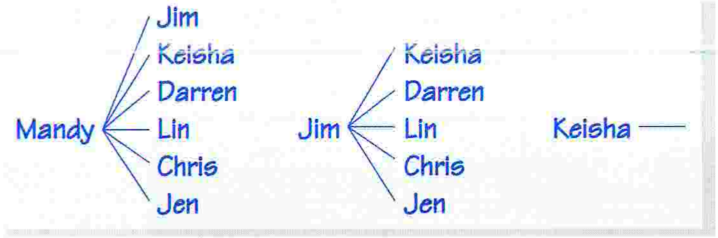
Plan and Solve

To make sure that you account for every pair of friends, make an organized list.

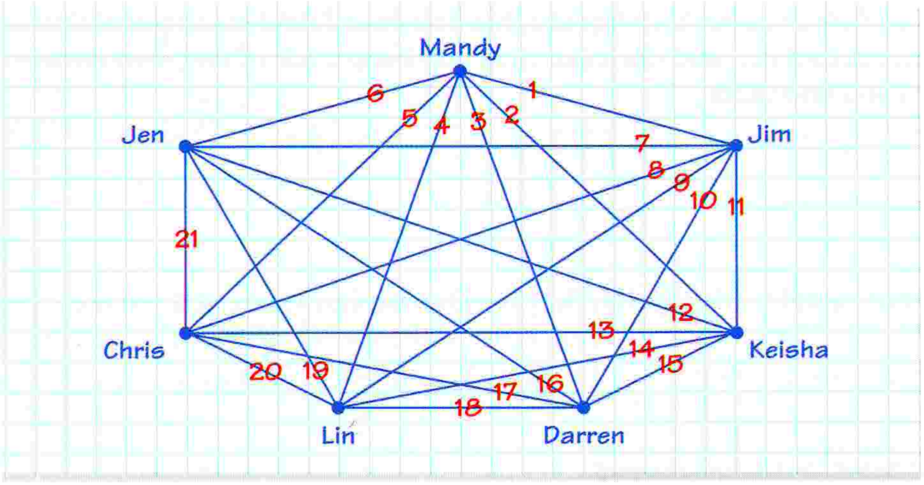
First pair Mandy with each of her six friends. Next, pair Jim with each of the five friends left. Since Mandy and Jim have already been paired, you don't need to count them again.



4. Copy and complete the list of paired friends.
5. What pattern do you see?
6. How many pictures do they need to take?

Look Back and Check

Another way to solve this problem is to use a diagram. Draw line segments to show all possible pairs of friends.



There are 21 line segments. This shows there are 21 pairs of friends.

✓ Check Understanding

7. Suppose Mandy and nine friends pair up for pictures. Use the pattern suggested above and find how many pictures there will be.

EXERCISES

For more exercises, see *Extra Practice.*

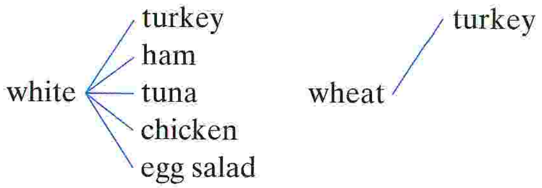
Practice and Problem Solving

A Practice by Example

Example 1 (page 197)

Solve each problem by accounting for all possibilities.

1. A sandwich shop serves turkey, ham, tuna, chicken, and egg salad sandwiches. You can have any sandwich using white, wheat, or rye bread. Suppose you eat there every day. For how many days can you order a sandwich that is different from any you have ordered before? The start of an organized list is shown above. Copy and complete the list to solve the problem.

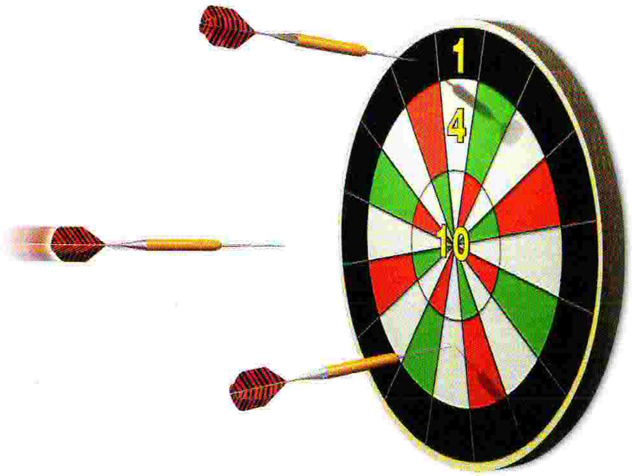


Strategies

- Account for All Possibilities
- Draw a Diagram
- Look for a Pattern
- Make a Model
- Make a Table
- Simplify the Problem
- Simulate the Problem
- Solve by Graphing
- Try, Test, Revise
- Use Multiple Strategies
- Work Backward
- Write an Equation
- Write a Proportion

2. You throw three darts at the board shown at the right. If each dart hits the board, what possible point totals can you score?

3. You have pepperoni, mushrooms, onions, and green peppers. How many different pizzas can you make by using one, two, three, or four of the toppings?



A dart landing on the board scores 1, 4, or 10 points.

4. **Elections** Four candidates run for president of the student council. Three other candidates run for vice-president. In how many different ways can the two offices be filled?

B Apply Your Skills

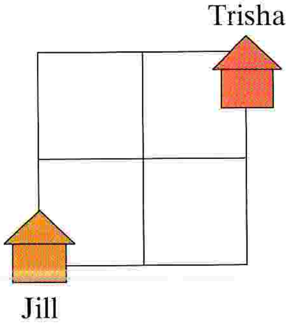
5. **Patterns** Eight people are at a party. Everyone shakes hands once with everyone else. How many handshakes are there altogether?

6. **Geometry** You have 24 feet of fence to make a rectangular garden. Each side will measure a whole number of feet. How many different-sized rectangular gardens can you make?

Solve using any strategy.

7. **Patterns** The bottom row of a stack of blocks contains 11 blocks. The row above it contains 9 blocks. The next higher row contains 7 blocks. The rows continue in this pattern, and the top row contains a single block. How many blocks does the stack contain in all?

C Challenge



8. **Routes** Copy the diagram at the left. Using the paths shown, Jill can walk to Trisha's house in many different ways. Draw each route that is four blocks long. How can you be sure that you have found all possible routes?

9. **Seating Arrangements** Ana, Brian, Carla, David, and Eric are friends. They go to a movie, but cannot find five seats together. They have to split up into a group of three and a group of two. How many different ways can the friends organize themselves into these two groups?

10. You have one penny, one nickel, one dime, and one quarter. How many different amounts of money can you make using one or more of these coins?

11. **Softball** There are seven softball teams in a league. Each team plays each of the other teams twice. What is the total number of games played?

12. **Geometry** How many different rectangles are there with an area of 36 cm^2 if the side lengths of each, in centimeters, are whole numbers?

Test Prep

Multiple Choice

13. Which list shows all the positive factors of 32?
   - A. 1, 2, 3, 8, 16, 32
   - B. 1, 2, 4, 8, 16, 32
   - C. 1, 2, 3, 4, 8, 16, 32
   - D. 1, 2, 4, 6, 8, 16, 32

14. What is the value of $(0.3)^2$?
   - F. 0.6
   - G. 0.9
   - H. 0.06
   - I. 0.09

15. What is the GCF of $24x^2y^3$ and $32x^3y$?
   - A. $4xy$
   - B. $4x^2y$
   - C. $8xy$
   - D. $8x^2y$

16. Which fraction is the simplest form of $\frac{45}{60}$?
   - F. $\frac{2}{3}$
   - G. $\frac{3}{4}$
   - H. $\frac{15}{20}$
   - I. $\frac{5}{6}$

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Mixed Review

Lesson 4-4 **Write in simplest form.**

17. $\frac{6}{12}$ 18. $\frac{10}{40}$ 19. $\frac{6a^2}{15}$ 20. $\frac{14a^3}{28a^2}$

Lesson 1-7 **Write a rule for each pattern.**

21. 10, 20, 30, . . . 22. 8, 5, 2, −1, . . . 23. 2, 6, 18, 54, . . .

Lesson 1-2

24. Elki has read the first 60 pages of a book. When he has read 35 more pages, he will have read half the book. How many pages are in the book?

Rational Numbers

4-6

What You'll Learn

OBJECTIVE 1 To identify and graph rational numbers

OBJECTIVE 2 To evaluate fractions containing variables

. . . And Why

To solve real-world problems involving rates

✓ Check Skills You'll Need

Write in simplest form.

1. $\frac{2}{10}$
2. $\frac{14}{21}$
3. $\frac{28}{35}$
4. $\frac{6}{8}$

For help, go to Lesson 4-4.

New Vocabulary

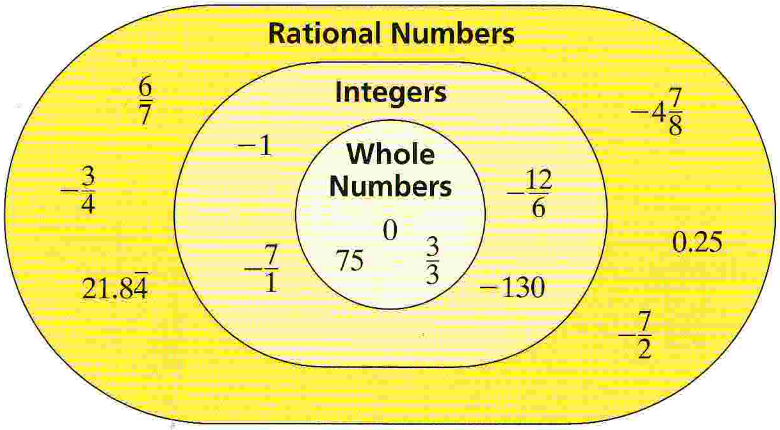
- **rational number**

OBJECTIVE

1 Identifying and Graphing Rational Numbers

A **rational number** is any number you can write as a quotient $\frac{a}{b}$ of two integers, where b is not zero. The diagram below shows relationships among rational numbers.

Notice that all integers are rational numbers. This is true because you can write any integer a as $\frac{a}{1}$.



Here are three ways you can write a negative rational number.

$$-\frac{7}{9} = \frac{-7}{9} = \frac{7}{-9}$$

For each rational number, there is an unlimited number of equivalent fractions.

1 EXAMPLE Writing Equivalent Fractions

Write two lists of fractions equivalent to $\frac{1}{2}$.

$\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \cdots$ **Numerators and denominators are positive.**

$\frac{1}{2} = \frac{-1}{-2} = \frac{-2}{-4} = \cdots$ **Numerators and denominators are negative.**

Need Help?

The quotient of two integers with the same sign is positive.

✓ Check Understanding Example 1

1. Write three fractions equivalent to each fraction.

a. $\frac{1}{3}$ b. $-\frac{4}{5}$ c. $\frac{5}{8}$ d. $-\frac{1}{2}$

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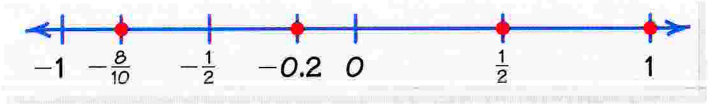
You can graph rational numbers on a number line.

2 EXAMPLE Graphing a Rational Number

Graph each rational number on a number line.

a. $\frac{1}{2}$ **b.** $-\frac{8}{10}$ **c.** 1 **d.** -0.2



Check Understanding Example 2

2. Graph each rational number on a number line.

a. $-\frac{1}{2}$ **b.** $-\frac{4}{10}$ **c.** -2 **d.** 0.9

OBJECTIVE

2 Evaluating Fractions Containing Variables

Recall that a fraction bar is a grouping symbol, so you first simplify the numerator and the denominator. Then, simplify the fraction.

Simplify the numerator. → **Simplify the denominator.** → $\frac{1 + 9 + 2}{2 - 5} = \frac{12}{-3} = -4 \leftarrow$ simplest form

To simplify a fraction with variables, first substitute for the variables.

3 EXAMPLE Real-World Problem Solving

Science The speed of a car changes from 37 ft/s to 102 ft/s in five seconds. What is its acceleration in feet per second per second (ft/s^2)? Use the formula $a = \frac{f - i}{t}$ where a is acceleration, f is final speed, i is initial speed, and t is time.

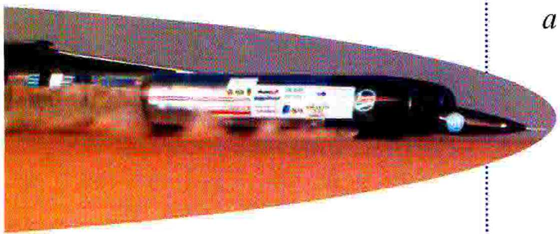
$a = \frac{f - i}{t}$ **Use the acceleration formula.**

$= \frac{102 - 37}{5}$ **Substitute for the variables.**

$= \frac{65}{5}$ **Subtract.**

$= 13$ **Write in simplest form.**

The car's acceleration is 13 ft/s^2.



Real-World Connection

The world's fastest car, the *Thrust SSC*, can go from 0 ft/s to 1,119 ft/s in thirty seconds.

Check Understanding Example 3

3. Evaluate for $a = 6$ and $b = -5$. Write in simplest form.

a. $\frac{a + b}{-3}$ **b.** $\frac{7 - b}{3a}$ **c.** $\frac{a + 9}{b}$

EXERCISES

For more exercises, see *Extra Practice*.

Practice and Problem Solving

A Practice by Example

Example 1 (page 201)

Show the next three fractions equivalent to $\frac{1}{2}$ in each list.

1. $\frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \ldots$
2. $\frac{1}{2}, \frac{-1}{-2}, \frac{-3}{-6}, \ldots$

Write three fractions equivalent to each fraction.

3. $\frac{1}{6}$
4. $\frac{3}{5}$
5. $-\frac{5}{9}$
6. $-\frac{4}{4}$
7. $-\frac{2}{3}$
8. $\frac{4}{7}$

Example 2 (page 202)

Graph each rational number on a number line.

9. $\frac{1}{10}$
10. $-\frac{3}{5}$
11. 2
12. -0.3
13. -0.75
14. $\frac{2}{3}$

Example 3 (page 202)

Evaluate for $a = -4$ and $b = -6$. Write in simplest form.

15. $\frac{a}{b}$
16. $\frac{a + 9}{b}$
17. $\frac{b + a}{3a}$
18. $\frac{2a + b}{20}$
19. $\frac{b + 7}{2a}$
20. $\frac{b - a}{3b}$

21. Boat Races The speed of a racing boat changes from 0 ft/s to 264 ft/s in six seconds. What is the acceleration of the boat in ft/s^2? Use the acceleration formula given in Example 3.

B Apply Your Skills

Write three fractions equivalent to each fraction.

22. $\frac{2}{8}$
23. $-\frac{2}{5}$
24. $\frac{4}{12}$
25. $-\frac{12}{27}$
26. $\frac{7}{11}$
27. $-\frac{5}{13}$

Write a rational number that is between each pair of numbers.

28. $0, -1$
29. $0.9, 1.1$
30. $\frac{-1}{5}, \frac{-4}{5}$

Evaluate. Write in simplest form.

31. $\frac{y}{-x}$, for $x = 5$ and $y = -4$
32. $\frac{-2y}{x^2}$, for $x = 9$ and $y = 3$
33. $\frac{m}{n}$, for $m = -2$ and $n = 8$
34. $\frac{m - n}{-12}$, for $m = -3$ and $n = 6$
35. $\frac{6b - 16}{3c}$, for $b = 8$ and $c = 12$
36. $\frac{3m - 11}{n}$, for $m = 7$ and $n = 14$

37. Which of the following rational numbers are equivalent to $-\frac{4}{5}$?
$\frac{4}{-5}, \frac{-12}{15}, -\frac{16}{20}, \frac{-4}{-5}$

38. a. Open-Ended Write two rational numbers between 0 and $\frac{1}{2}$.
b. How many other rational numbers are between 0 and $\frac{1}{2}$? Explain.

39. Reasoning What are three fractions equivalent to $\frac{a}{b}$? Explain.

40. Science The formula $s = \frac{1{,}600}{d^2}$ gives the strength s of a radio signal at a distance d miles from the transmitter. What is the strength at 5 mi? Write your answer in simplest form.

41. Writing in Math Explain why a whole number is an integer and an integer is a rational number.

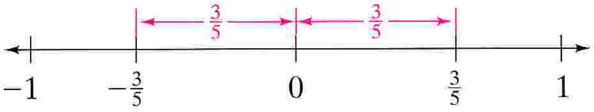
42. If the *Thrust SSC* (see page 202) can go from 0 ft/s to 1,119 ft/s in 30 s, what is its acceleration in feet per second per second?

C Challenge

Write the opposite and the absolute value of each number.

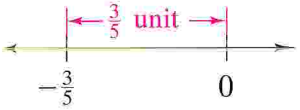
SAMPLE Find the opposite and the absolute value of $-\frac{3}{5}$.

Opposite:



$-\frac{3}{5}$ and $\frac{3}{5}$ are opposites.

Absolute value:



$\left|-\frac{3}{5}\right| = \frac{3}{5}$

43. $\frac{2}{3}$ **44.** $-\frac{5}{6}$ **45.** $\frac{-4}{5}$ **46.** $\frac{2}{-7}$ **47.** $\frac{-3}{-5}$ **48.** $-\frac{a}{b}$

Reasoning **Tell whether each statement is true for all positive integers *a* and *b*. If the statement is not always true, give a counterexample.**

49. $\frac{a^2}{b} > \frac{a}{b}$ **50.** $\frac{3a}{3b} = \frac{a}{b}$ **51.** $\frac{a^2}{b^2} > \frac{a}{b}$

Test Prep

Multiple Choice

52. Which three fractions are equivalent to $\frac{3}{4}$?

A. $\frac{6}{8}, \frac{9}{16}, \frac{-6}{-8}$ **B.** $\frac{-3}{4}, \frac{12}{16}, \frac{15}{20}$ **C.** $\frac{6}{8}, \frac{-3}{-4}, \frac{9}{12}$ **D.** $\frac{15}{20}, \frac{12}{16}, \frac{-6}{8}$

53. What is the simplest form of $\frac{y(xy - 7)}{10}$ when $x = 6$ and $y = 2$?

F. 3 **G.** $\frac{30}{10}$ **H.** 1 **I.** $\frac{10}{10}$

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54. Which pair of numbers is between −3 and −2?

A. $-2\frac{1}{2}, -2\frac{1}{3}$ **B.** $-3\frac{1}{2}, -3\frac{1}{3}$ **C.** $-2, -\pi$ **D.** $-2\frac{1}{2}, -3$

Short Response

55. Is $\frac{a^2}{b^2} > \frac{a}{b}$ true for all negative integers *a* and *b*? Explain.

Mixed Review

Lesson 4-5

56. **Patterns** Lucia has 4 pairs of slacks, 5 shirts, and 2 sweaters. How many different three-piece outfits can she make?

Lesson 1-9

Multiply or divide.

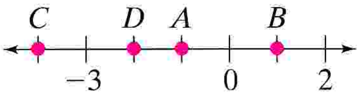
57. $-7 \cdot 4$ **58.** $19(-5)$ **59.** $-124 \div (-4)$ **60.** $-204 \div 6$

Lesson 1-4

Write the integer represented by each point on the number line.

61. *A* **62.** *B*

63. *C* **64.** *D*



Exponents and Multiplication

4-7

OBJECTIVE 1 Multiplying Powers With the Same Base

In Lesson 4-2, you learned how to use exponents to indicate repeated multiplication. What happens when you multiply two powers with the same base, such as 7^2 and 7^3?

$$7^2 \cdot 7^3 = (7 \cdot 7) \cdot (7 \cdot 7 \cdot 7) = 7^5$$

Notice that $7^2 \cdot 7^3 = 7^5 = 7^{2+3}$. In general, when you multiply powers with the same base, you can add the exponents.

Key Concepts Multiplying Powers With the Same Base

To multiply numbers or variables with the same base, add the exponents.

Arithmetic	**Algebra**
$2^3 \cdot 2^4 = 2^{3+4} = 2^7$	$a^m \cdot a^n = a^{m+n}$, for positive integers m and n.

What You'll Learn

- OBJECTIVE 1 To multiply powers with the same base
- OBJECTIVE 2 To find a power of a power

. . . And Why

To learn the rules for operating with exponents

✓ Check Skills You'll Need

Write using exponents.

1. $k \cdot k \cdot k \cdot k$
2. $m \cdot n \cdot m \cdot n$
3. $2 \cdot 2 \cdot 2 \cdot 2$
4. $5 \cdot 5 \cdot 5$

For help, go to Lesson 4-2.

You *simplify* an expression by doing as many of the indicated operations as possible.

1 EXAMPLE Multiplying Powers

Simplify each expression.

a. $3 \cdot 3^3$

$3^1 \cdot 3^3 = 3^{1+3}$ **Add the exponents of powers with the same base.**

$= 3^4$

$= 81$ **Simplify.**

b. $a^5 \cdot a \cdot b^2$

$a^5 \cdot a^1 \cdot b^2 = a^{5+1}b^2$ **Add the exponents of powers with the same base.**

$= a^6b^2$ **Simplify.**

Need Help?

Recall that $3 = 3^1$ and $a = a^1$ because a base with exponent 1 is equal to the base itself.

✓ Check Understanding Example 1

1. Simplify each expression.

   a. $2^2 \cdot 2^3$ b. $m^5 \cdot m^7$ c. $x^2 \cdot x^3 \cdot y \cdot y^4$

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2 EXAMPLE Using the Commutative Property

Simplify $-2x^2 \cdot 3x^5$.

$-2x^2 \cdot 3x^5 = -2 \cdot 3 \cdot x^2 \cdot x^5$ **Use the Commutative Property of Multiplication.**

$= -6x^{2+5}$ **Add the exponents.**

$= -6x^7$ **Simplify.**

Test-Taking Tip

When in doubt, write it out! If you are unsure about the rules for multiplying powers, write the powers out. For instance, write $x^2 \cdot x^5$ as $(x \cdot x) \cdot (x \cdot x \cdot x \cdot x \cdot x)$. This simplifies to x^7.

✓ Check Understanding Example 2

2. Simplify each expression.

a. $6a^3 \cdot 3a$ **b.** $-5c^2 \cdot -3c^7$ **c.** $4x^2 \cdot 3x^4$

OBJECTIVE

2 Finding a Power of a Power

You can find the power of a power by using the rule of Multiplying Powers With the Same Base.

$(7^2)^3 = (7^2) \cdot (7^2) \cdot (7^2)$ **Use 7^2 as a base 3 times.**

$= 7^{2+2+2}$ **When multiplying powers with the same base, add the exponents.**

$= 7^6$ **Simplify.**

Notice that $(7^2)^3 = 7^6 = 7^{2 \cdot 3}$. You can raise a power to a power by multiplying the exponents.

Key Concepts Finding a Power of a Power

To find a power of a power, multiply the exponents.

Arithmetic

$(2^3)^4 = 2^{3 \cdot 4} = 2^{12}$

Algebra

$(a^m)^n = a^{m \cdot n}$, for positive integers m and n.

3 EXAMPLE Simplifying Powers of Powers

Simplify each expression.

a. $(3^2)^3$

$(3^2)^3 = (3)^{2 \cdot 3}$ ← **Multiply the exponents.**

$= (3)^6$ ← **Simplify the exponent.**

$= 729$ ← **Simplify.**

b. $(a^6)^2$

Multiply the exponents. → $(a^6)^2 = a^{6 \cdot 2}$

Simplify the exponent. → $= a^{12}$

Reading Math

You read $(3^2)^3$ as "three squared to the third power." You read $(a^6)^2$ as "*a* to the sixth power squared."

✓ Check Understanding Example 3

3. Simplify each expression.

a. $(2^4)^2$ **b.** $(c^5)^4$ **c.** $(m^3)^2$

EXERCISES

For more exercises, see *Extra Practice.*

Practice and Problem Solving

A Practice by Example

Simplify each expression.

Example 1 (page 205)

1. $4^2 \cdot 4$

2. $a^2 \cdot a^5$

3. $x^4 \cdot y \cdot x^5 \cdot y$

4. $10^2 \cdot 10^5$

5. $2^2 \cdot 2^5$

6. $x^4 \cdot x^4$

7. $m^{50} \cdot m^2$

8. $(3)^2 \cdot (2)^3 \cdot 2 \cdot 3$

9. $x \cdot y \cdot y \cdot x^5 \cdot y^3$

Example 2 (page 206)

10. $7b^3 \cdot 4b^4$

11. $-9c^2 \cdot -2c^8$

12. $5x^3 \cdot 2x^6$

13. $4y^7 \cdot 6y^4$

14. $-2a^2 \cdot -2a^2$

15. $9b^2 \cdot -4b^2$

16. $-7x^6 \cdot -5x^8$

17. $-5d^5 \cdot 6d^2$

18. $4b^4 \cdot 12b^7$

Example 3 (page 206)

19. $(10^3)^2$

20. $(x^3)^4$

21. $(m^6)^4$

22. $(2^2)^3$

23. $(3^2)^4$

24. $(c^2)^8$

25. $(x^5)^7$

26. $(0^5)^8$

27. $(g^8)^{12}$

B Apply Your Skills

Complete each equation.

28. $8^2 \cdot 8^{\blacksquare} = 8^9$

29. $c^{\blacksquare} \cdot c^4 = c^{11}$

30. $(9^{\blacksquare})^4 = 9^{16}$

31. $5^6 \cdot 5^{\blacksquare} = 5^{14}$

32. $x^{\blacksquare} \cdot x^{12} = x^{15}$

33. $(a^{\blacksquare})^9 = a^{27}$

Compare. Use >, <, or = to complete each statement.

34. $25^2 \;\blacksquare\; (5^2)^2$

35. $(2^7)^7 \;\blacksquare\; (2^{25})^2$

36. $(4^3 \cdot 4^2)^3 \;\blacksquare\; 4^9$

37. **Open-Ended** A megabyte is 2^{20} bytes. Use exponents to write 2^{20} in four different ways.

38. **Writing in Math** Explain why $x^8 \cdot x^2$ has the same value as $x^5 \cdot x^5$.

39. **Error Analysis** Marcos thinks that $x^4 + x^4$ simplifies to $2x^4$. Doug thinks that $x^4 + x^4$ simplifies to x^8. Which result is correct? Explain.

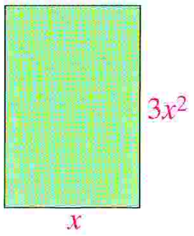
C Challenge

40. **Reasoning** Does $-(2^3)^2$ have the same value as $(-2^3)^2$? Justify your answer.

41. **Reasoning** Which of 2^{30} or 2^{16} is twice the value of 2^{15}? Explain.

Geometry **Find the area of each rectangle.**

42.



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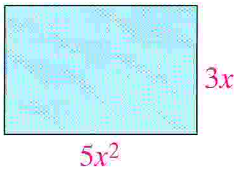

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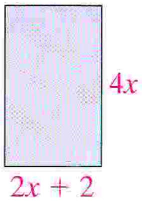

Test Prep

Multiple Choice

45. $(x^2)(y^5)(x) = \underline{\ ?\ }$

A. x^2y **B.** x^2y^5 **C.** x^3y^5 **D.** x^7y

46. What is the simplest form for $a^{10} \cdot a \cdot a^2$?

F. $a^{10} \cdot a^3$ **G.** a^{12} **H.** a^{13} **I.** a^{20}

47. $(9^5)^5 = \underline{\ ?\ }$

A. 9^1 **B.** 9^{10} **C.** 9^{25} **D.** 9^{55}

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Short Response

48. a. Find four expressions equivalent to 2^{13}.

b. Explain why each is equivalent to 2^{13} for part (a).

Mixed Review

Lesson 4-6 **Evaluate. Write in simplest form.**

49. $\frac{mn}{m - 6}$, for $m = 4$ and $n = 2$

50. $\frac{g + gh}{h - g}$, for $g = -3$ and $h = -5$

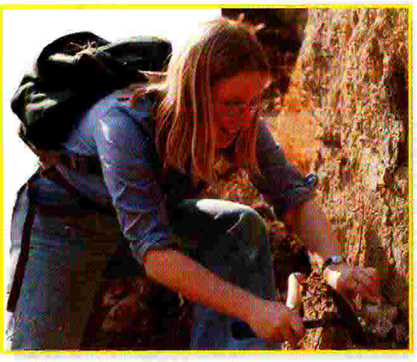
Lesson 2-8 **Graph the solutions of each inequality on a number line.**

51. $x < -3$ **52.** $a > 0$ **53.** $y \leq -4$ **54.** $b > -2$

Lesson 1-2 **55. Party Planning** The Scotts are getting ready for a barbeque. They buy 8 lb of hamburger at \$1.50/lb and 10 lb of chicken at \$1.25/lb. Write and simplify an expression that shows the total cost.

Math at Work

Geophysicist



A **geophysicist** studies Earth's surface, including the history of Earth's crust and rock formations. Geophysicists search for oil, natural gas, minerals, and underground water. They also work to solve environmental problems. They study what makes up Earth's interior, as well as its magnetic, electrical, and gravitational forces. They often study earthquakes and volcanoes.

Geophysicists use physics and mathematics in their studies. Much of their work involves measurement. They use instruments to track sound waves, gravity, energy waves, and magnetic fields. Exponents appear in the data that geophysicists gather because they often work with very large numbers.

Take It to the NET For more information about geophysicists, go to **www.PHSchool.com.**
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Technology

Evaluating Expressions With Graphing Calculators

For Use With Lesson 4-7

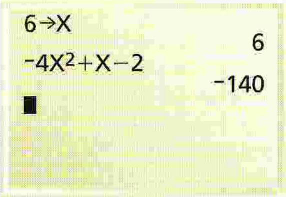
You can use a graphing calculator to evaluate an expression.

1 EXAMPLE

Use a graphing calculator to evaluate $-4x^2 + x - 2$ for $x = 6$.

Step 1 First store the value 6 to the variable x.
Press 6 STO▶ X ENTER.

Step 2 Enter $-4x^2 + x - 2$. Press ENTER.



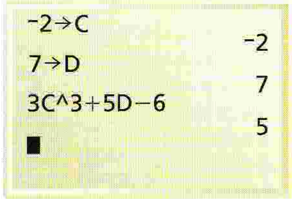
```
6→X
                6
-4X²+X-2
             -140
▮
```

To evaluate the expression in Example 1 for another value of x, start by storing that value to x. Press **ENTRY** *twice* to recall the expression $-4x^2 + x - 2$ from memory. Then press ENTER again. Repeat to evaluate the expression for another value of x.

2 EXAMPLE

Use a graphing calculator to evaluate $3c^3 + 5d - 6$ for $c = -2$ and $d = 7$.

You can evaluate an expression with more than one variable by following Step 1 twice to store the value of each variable separately. Then follow the directions in Step 2 to enter and evaluate the expression.



```
-2→C
               -2
7→D
                7
3C^3+5D-6
                5
▮
```

To evaluate the expression for other values of the variables, first store the new values. Press **ENTRY** until the expression appears on the calculator screen. Then press ENTER again to evaluate the expression.

EXERCISES

Use a graphing calculator. Evaluate each expression for the given values of the variable(s). Round to the nearest hundredth where necessary.

1. $x^2 - 3x + 8$
a. $x = 2$ **b.** $x = -3$ **c.** $x = 4.2$

2. $y^6 + y$
a. $y = -5$ **b.** $y = 3$ **c.** $y = 1.5$

3. $-4c^2 + 34b - 42$
a. $c = -12; b = 3$ **b.** $c = 2; b = 4$ **c.** $c = 5.1; b = 4$

4. Evaluate $6x^3 - 2x$ for whole number values of x from 1 to 10.

4-8 Exponents and Division

What You'll Learn

OBJECTIVE 1 To divide expressions containing exponents

OBJECTIVE 2 To simplify expressions with integer exponents

. . . And Why

To solve real-world problems involving science

✓ Check Skills You'll Need

Write in simplest form.

1. $\frac{x^2}{x}$
2. $\frac{y}{y^2}$
3. $\frac{6xy}{9y}$
4. $\frac{4ab^2}{16b}$

For help, go to Lesson 4-4.

OBJECTIVE

1 Dividing Expressions Containing Exponents

In Lesson 4-7, you learned that you add exponents to multiply powers with the same base. To divide powers with the same base, you subtract exponents. Here's why.

$$\frac{7^8}{7^3} = \frac{7\cdot7\cdot7\cdot7\cdot7\cdot7\cdot7\cdot7}{7\cdot7\cdot7}$$ **Expand the numerator and denominator.**

$$= \frac{\cancel{7}^1\cdot\cancel{7}^1\cdot\cancel{7}^1\cdot7\cdot7\cdot7\cdot7\cdot7}{{}_1\cancel{7}\cdot{}_1\cancel{7}\cdot{}_1\cancel{7}}$$ **Divide common factors.**

$$= 7^5$$

Notice that $\frac{7^8}{7^3} = 7^5 = 7^{8-3}$. This suggests the following rule.

Key Concepts Dividing Powers With the Same Base

To divide numbers or variables *with the same nonzero base,* subtract the exponents.

Arithmetic	**Algebra**
$\frac{4^5}{4^2} = 4^{5-2} = 4^3$	$\frac{a^m}{a^n} = a^{m-n}$, for $a \neq 0$ and positive integers m and n.

1 EXAMPLE Dividing a Power by a Power

Simplify each expression.

a. $\frac{3^8}{3^5}$ **b.** $\frac{a^4}{a^2}$

$\frac{3^8}{3^5} = 3^{8-5}$ ⟵ **Subtract the exponents.** ⟶ $\frac{a^4}{a^2} = a^{4-2}$

$= 3^3$ ⟵ **Simplify the exponent.** ⟶ $= a^2$

$= 27$ ⟵ **Simplify.**

✓ Check Understanding Example 1

1. Simplify each expression.

a. $\frac{10^7}{10^4}$ **b.** $\frac{x^{25}}{x^{18}}$ **c.** $\frac{12m^5}{3m}$

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OBJECTIVE

2 Simplifying Expressions With Integer Exponents

What happens when you divide powers with the same base and get zero as an exponent? Consider $\frac{3^4}{3^4}$.

$$\frac{3^4}{3^4} = 3^{4-4} = 3^0 \qquad \frac{3^4}{3^4} = \frac{\cancel{3}^1 \cdot \cancel{3}^1 \cdot \cancel{3}^1 \cdot \cancel{3}^1}{{}_1\cancel{3} \cdot {}_1\cancel{3} \cdot {}_1\cancel{3} \cdot {}_1\cancel{3}} = \frac{1}{1} = 1$$

Notice that $\frac{3^4}{3^4} = 3^0$ and $\frac{3^4}{3^4} = 1$. This suggests the following rule.

Key Concepts Zero as an Exponent

Arithmetic	Algebra
$3^0 = 1$	$a^0 = 1$, for $a \neq 0$.

2 EXAMPLE Simplifying When Zero Is an Exponent

Simplify each expression.

a. $\frac{(-8)^2}{(-8)^2}$

$\frac{(-8)^2}{(-8)^2} = (-8)^{2-2}$ **Subtract the exponents.**

$= (-8)^0$ **Simplify.**

$= 1$

b. $\frac{6b^3}{18b^3}$

$\frac{6b^3}{18b^3} = \frac{1}{3}b^0$ **Subtract the exponents. Simplify $\frac{6}{18}$.**

$= \frac{1}{3} \cdot 1$ **Simplify b^0.**

$= \frac{1}{3}$ **Multiply.**

Check Understanding Example 2

2. Simplify each expression.

a. 43^0 **b.** $\frac{5^2x^6}{5x^6}$ **c.** $\frac{x^5y^6}{x^5y^3}$ **d.** $5x^0$

What happens when you divide powers with the same base and get a negative exponent? Consider $\frac{3^2}{3^4}$.

$$\frac{3^2}{3^4} = 3^{2-4} = 3^{-2} \qquad \frac{3^2}{3^4} = \frac{\cancel{3}^1 \cdot \cancel{3}^1}{{}_1\cancel{3} \cdot {}_1\cancel{3} \cdot 3 \cdot 3} = \frac{1}{3^2}$$

These results suggest the rule at the top of page 212.

Key Concepts Negative Exponents

Arithmetic	**Algebra**
$3^{-2} = \frac{1}{3^2}$	$a^{-n} = \frac{1}{a^n}$, for $a \neq 0$.

Real-World Connection

Hummingbirds may range from 0.0022 kg to 0.02 kg in mass.

A hummingbird has a mass of about 10^{-2} kg, or $\frac{1}{10^2}$ kg. To simplify 10^{-2}, you write $\frac{1}{100}$ or 0.01. So the hummingbird has a mass of 0.01 kg. To simplify an expression such as x^{-2}, you write it as $\frac{1}{x^2}$, using no negative exponents.

3 EXAMPLE Using Positive Exponents

Simplify each expression.

a. $\frac{5^6}{5^8}$ **b.** $\frac{m^2}{m^5}$

$\frac{5^6}{5^8} = 5^{6-8}$ ← **Subtract the exponents.** → $\frac{m^2}{m^5} = m^{2-5}$

$= 5^{-2}$ $= m^{-3}$

$= \frac{1}{5^2}$ ← **Write with a positive exponent.** → $= \frac{1}{m^3}$

$= \frac{1}{25}$ ← **Simplify.**

Check Understanding Example 3

3. Simplify each expression.

a. $\frac{4^5}{4^7}$ **b.** $\frac{a^4}{a^6}$ **c.** $\frac{3y^8}{9y^{12}}$

You can also write an expression such as $\frac{1}{x^2}$ so that there is no fraction bar.

4 EXAMPLE Using Negative Exponents

Write $\frac{x^2y^3}{x^3y}$ without a fraction bar.

$\frac{x^2y^3}{x^3y} = x^{2-3}y^{3-1}$ **Use the Rule for Dividing Powers With the Same Base.**

$= x^{-1}y^2$ **Subtract the exponents.**

Check Understanding Example 4

4. Write each expression without a fraction bar.

a. $\frac{b^3}{b^9}$ **b.** $\frac{m^3n^2}{m^6n^8}$ **c.** $\frac{xy^5}{x^5y^3}$

EXERCISES

For more exercises, see *Extra Practice*.

Practice and Problem Solving

A Practice by Example

Simplify each expression.

Example 1 (page 210)

1. $\frac{2^5}{2^2}$
2. $\frac{h^6}{h^2}$
3. $\frac{10y^7}{6y^2}$
4. $\frac{10b^8}{2b^6}$
5. $\frac{6^2}{6^1}$
6. $\frac{11^5}{11^3}$
7. $\frac{x^7}{x^3}$
8. $\frac{a^{27}}{a^{19}}$

Example 2 (page 211)

9. $\frac{18x^{20}}{18x^{20}}$
10. $(-4)^0$
11. $\frac{w^8z^{15}}{w^8z^8}$
12. 3^0
13. $\frac{b^3c^2}{b^3c}$
14. $\frac{(-2)^4}{(-2)^4}$
15. $2b^0$
16. $\frac{2y^3}{8y^3}$

Example 3 (page 212)

17. $\frac{7^3}{7^5}$
18. $\frac{m^2}{m^6}$
19. $\frac{4a^3}{20a^6}$
20. $\frac{100m^{100}}{200m^{200}}$
21. $\frac{b^5}{b^8}$
22. $\frac{3m}{15m^3}$
23. $\frac{6^7}{6^{11}}$
24. $\frac{a^2}{a^7}$

Example 4 (page 212)

Write each expression without a fraction bar.

25. $\frac{y^4}{y^7}$
26. $\frac{a^2b^4}{a^8b^2}$
27. $\frac{m^5n^6}{m^7n^8}$
28. $\frac{xy^2}{x^4y^9}$

B Apply Your Skills

Complete each equation.

29. $\frac{x^6}{x^{\blacksquare}} = x^4$
30. $\frac{14x^5}{7x^3} = 2x^{\blacksquare}$
31. $\frac{10^5}{10^{\blacksquare}} = 1$
32. $\frac{1}{a^3} = a^{\blacksquare}$
33. $\frac{y^{\blacksquare}}{y^9} = y^{-4}$
34. $\frac{1}{-27} = (-3)^{\blacksquare}$

Real-World Connection

The photo shows damage from the Loma Prieta, California, earthquake of October 17, 1989.

35. **Earthquakes** The *magnitude* of an earthquake is a measure of the amount of energy released. An earthquake of magnitude 6 releases about 30 times as much energy as an earthquake of magnitude 5. The magnitude of the 1989 earthquake in Loma Prieta, California, was about 7. The magnitude of the 1933 earthquake in Sanriku, Japan, was about 9. Simplify $\frac{30^9}{30^7}$ to find how many times as much energy was released in the Sanriku earthquake.

36. **Error Analysis** A student wrote that $-5^0 = 1$. What was the student's error?

37. **Open-Ended** Write three different quotients that equal 5^{-7}.

38. **Writing in Math** Is -3^{-2} positive or negative? Justify your answer.

Write each expression without a fraction bar.

39. $\frac{x^3}{x^5}$
40. $\frac{a^9b^3}{a^7b^8}$
41. $\frac{m^9n^3}{m^2n^{10}}$
42. $\frac{b^{14}c^2}{b^9c^{11}}$

C Challenge

Simplify each expression.

43. $\frac{5x^2}{10x^{-5}}$
44. $\frac{5b^{-7}}{5b^{-2}}$
45. $\frac{4^2 + 6^2}{2^2}$
46. $\frac{r^{-5}}{s^{-2}}$

Test Prep

Multiple Choice

47. What is a simpler form of $\frac{x^5y^4}{x^2y^9}$?

A. $\frac{x^5}{y^3}$ **B.** $\frac{y^5}{x^3}$ **C.** x^3y^5 **D.** $\frac{x^3}{y^5}$

48. What is a simpler form of $\frac{12a^{35}}{36a^{50}}$?

F. $\frac{1^{85}}{3^a}$ **G.** $\frac{1}{3a^{85}}$ **H.** $\frac{1}{3a^{15}}$ **I.** $\frac{1}{3}a^{15}$

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49. Which expression is equal to $\frac{42a^6b^7}{7a^3b^3}$?

A. $6a^3b^{-4}$ **B.** $6a^3b^4$ **C.** $6a^{-3}b^4$ **D.** $6a^{-3}b^{-4}$

Short Response

50. **a.** Is 5^{-3} a negative number? **b.** Explain your answer.

Mixed Review

Lesson 4-7

Simplify each expression.

51. $5^2 \cdot 5$ **52.** $x^7 \cdot x^2$ **53.** $2a^9 \cdot 8a^7$

Lesson 3-1

Estimate using front-end estimation.

54. 5.68 + 3.24 **55.** 17.86 + 2.321 **56.** 20.2 + 5.8

Lesson 2-7

57. **Number Sense** The sum of three consecutive integers is 264. What are the three integers?

Checkpoint Quiz 2 Lessons 4-5 through 4-8

TEXT Instant self-check quiz online and on CD-ROM

Write three fractions equivalent to each given fraction.

1. $\frac{3}{12}$ **2.** $\frac{12}{36}$ **3.** $\frac{49}{70}$ **4.** $\frac{18}{28}$ **5.** $\frac{4}{5}$

Evaluate for $a = 4$ and $b = -6$. Write in simplest form.

6. $\frac{a}{2b}$ **7.** $\frac{b+a}{a}$ **8.** $\frac{a-b}{15}$ **9.** $\frac{b-a}{a^2}$ **10.** $\frac{3a+b}{24}$

Graph the rational numbers below on the same number line.

11. -0.8 **12.** $\frac{1}{2}$ **13.** 0.6 **14.** $-\frac{2}{10}$ **15.** $\frac{9}{10}$

Simplify each expression.

16. $2^3 \cdot 2^4$ **17.** $(x^5)^{10}$ **18.** $\frac{18a^4}{3a^2}$ **19.** $\frac{x^3}{x^8}$ **20.** $\frac{a^3b^5}{a^9b^5}$

21. If 12 of 16 students vote to do a project, what fraction of the students is this? Write the fraction in simplest form.

Scientific Notation

4-9

OBJECTIVE

1 Writing and Evaluating Scientific Notation

Investigation

Exploring Scientific Notation

1. Copy and complete the chart below.

5×10^4	$= 5 \times 10{,}000$	$= 50{,}000$	
5×10^3	$= 5 \times 1{,}000$	$=$ ■	
5×10^2	$= 5 \times$ ■	$=$ ■	
5×10^1	$= 5 \times$ ■	$=$ ■	
5×10^0	$= 5 \times$ ■	$=$ ■	
5×10^{-1}	$= 5 \times \frac{1}{10}$	$= 5 \times 0.1$	$= 0.5$
5×10^{-2}	$= 5 \times$ ■	$= 5 \times 0.01$	$= 0.05$
5×10^{-3}	$= 5 \times$ ■	$= 5 \times$ ■	$= 0.005$
5×10^{-4}	$= 5 \times$ ■	$= 5 \times$ ■	$=$ ■

2. **Patterns** Describe any related patterns that you see in your chart.

3. **a.** Based on the patterns you see, simplify 5×10^7.
b. Simplify 5×10^{-6}.

Scientific notation provides a way to write numbers using powers of 10. You write a number in scientific notation as the product of two factors.

Second factor is a power of 10.

$$7{,}500{,}000{,}000{,}000 = 7.5 \times 10^{12}$$

First factor is greater than or equal to 1, but less than 10.

Scientific notation lets you know the size of a number without having to count digits. For example, if the exponent of 10 is 6, the number is in the millions. If the exponent is 9, the number is in the billions.

What You'll Learn

OBJECTIVE 1 To write and evaluate numbers in scientific notation

OBJECTIVE 2 To calculate with scientific notation

. . . And Why

To solve real-world problems involving weight and mass

Check Skills You'll Need

Write each expression with a single exponent.

1. $10^3 \cdot 10^5$

2. $10^7 \cdot 10^9$

3. $10^5 \cdot 10^{-3}$

4. $10^{-6} \cdot 10^3$

For help, go to Lesson 4-7.

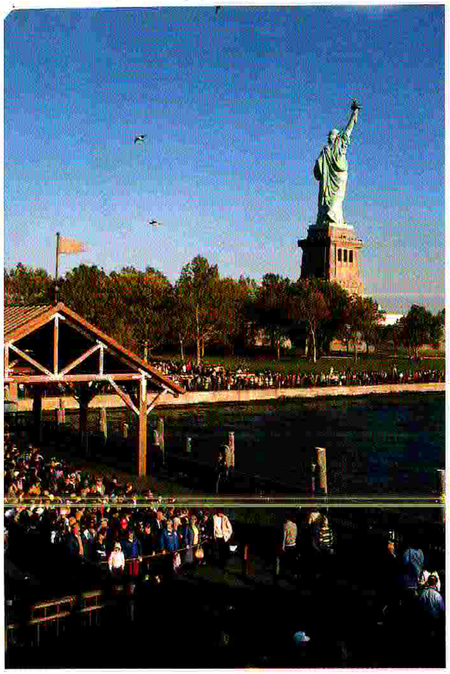
New Vocabulary

- scientific notation
- standard notation

Need Help?

For help with multiplying by powers of ten, see Skills Handbook, page 768.

iTEXT Interactive lesson includes instant self-check, tutorials, and activities.



Real-World Connection

The total weight of the Statue of Liberty is about 450,000 lb.

1 EXAMPLE Real-World Problem Solving

About 4,200,000 people visit the Statue of Liberty every year. Write this number in scientific notation.

$4{,}\underbrace{200{,}000}_{\text{6 places}}$ **Move the decimal point to get a decimal greater than 1 but less than 10.**

4.2 **Drop the zeros after the 2.**

4.2×10^{6} **You moved the decimal point 6 places. The number is large. Use 6 as the exponent of 10.**

Check Understanding Example 1

1. Write each number in scientific notation.

a. 54,500,000 **b.** 723,000 **c.** 602,000,000,000

In scientific notation, you use a negative exponent to write a number between 0 and 1.

2 EXAMPLE Writing in Scientific Notation

Write 0.000079 in scientific notation.

$0.\underbrace{00007}_{\text{5 places}}9$ **Move the decimal point to get a decimal greater than 1 but less than 10.**

7.9 **Drop the zeros before the 7.**

7.9×10^{-5} **You moved the decimal point 5 places. The number is small. Use −5 as the exponent of 10.**

Check Understanding Example 2

2. Write each number in scientific notation.

a. 0.00021 **b.** 0.00000005 **c.** 0.0000000000803

You can change expressions from scientific notation to **standard notation** by simplifying the product of the two factors.

3 EXAMPLE Writing in Standard Notation

Write each number in standard notation.

a. 8.9×10^{5} **b.** 2.71×10^{-6}

$8.\underbrace{90000}$ **Write zeros while moving the decimal point.** $\underbrace{000002}.71$

890,000 **Rewrite in standard notation.** 0.00000271

✓ Check Understanding Example 3

3. Write each number in standard notation.

a. 3.21×10^7 **b.** 5.9×10^{-8} **c.** 1.006×10^{10}

For a number to be in scientific notation, the digit in front of the decimal must be 1 or between 1 and 10.

4 EXAMPLE Changing to Scientific Notation

Write each number in scientific notation.

a. $\mathbf{0.37 \times 10^{10}}$

$0.37 \times 10^{10} = 3.7 \times 10^{-1} \times 10^{10}$ **Write 0.37 as 3.7×10^{-1}.**

$= 3.7 \times 10^9$ **Add the exponents.**

b. $\mathbf{453.1 \times 10^8}$

$453.1 \times 10^8 = 4.531 \times 10^2 \times 10^8$ **Write 453.1 as 4.531 x 10^2.**

$= 4.531 \times 10^{10}$ **Add the exponents.**

✓ Check Understanding Example 4

4. Write each number in scientific notation.

a. 16×10^5 **b.** 0.203×10^6 **c.** $7{,}243 \times 10^{12}$

You can compare and order numbers using scientific notation. First compare the powers of 10, and then compare the decimals.

5 EXAMPLE Comparing and Ordering Numbers

Order 0.064×10^8, 312×10^2, and 0.58×10^7 from least to greatest.

Write each number in scientific notation.

0.064×10^8 312×10^2 0.58×10^7

$\downarrow$ $\downarrow$ $\downarrow$

6.4×10^6 3.12×10^4 5.8×10^6

Order the powers of 10. Arrange the decimals with the same power of 10 in order.

3.12×10^4 5.8×10^6 6.4×10^6

Write the original numbers in order.

$312 \times 10^2, 0.58 \times 10^7, 0.064 \times 10^8$

✓ Check Understanding Example 5

5. Order from least to greatest.

a. $526 \times 10^7, 18.3 \times 10^6, 0.098 \times 10^9$
b. $8 \times 10^{-9}, 14.7 \times 10^{-7}, 0.22 \times 10^{-10}$

OBJECTIVE

2 Calculating With Scientific Notation

You can multiply numbers in scientific notation using the rule for Multiplying Powers with the Same Base.

6 EXAMPLE Multiplying With Scientific Notation

Multiply 3×10^{-7} and 9×10^{3}. Express the result in scientific notation.

$(3 \times 10^{-7})(9 \times 10^{3}) = 3 \times 9 \times 10^{-7} \times 10^{3}$ **Use the Commutative Property of Multiplication.**

$= 27 \times 10^{-7} \times 10^{3}$ **Multiply 3 and 9.**

$= 27 \times 10^{-4}$ **Add the exponents.**

$= 2.7 \times 10^{1} \times 10^{-4}$ **Write 27 as 2.7×10^{1}.**

$= 2.7 \times 10^{-3}$ **Add the exponents.**

✓ Check Understanding **Example 6**

6. Multiply. Express each result in scientific notation.

a. $(4 \times 10^{4})(6 \times 10^{6})$ **b.** $(7.1 \times 10^{-8})(8 \times 10^{4})$

7 EXAMPLE Real-World Problem Solving

Measurement **The Great Pyramid of Giza in Egypt contains about 2.3×10^{6} blocks of stone. On the average, each block of stone weighs about 5×10^{3} lb. About how many pounds of stone does the Great Pyramid contain?**

$(2.3 \times 10^{6})(5 \times 10^{3})$ **Multiply number of blocks by weight of each.**

$= 2.3 \times 5 \times 10^{6} \times 10^{3}$ **Use the Commutative Property of Multiplication.**

$= 11.5 \times 10^{6} \times 10^{3}$ **Multiply 2.3 and 5.**

$= 11.5 \times 10^{9}$ **Add the exponents.**

$= 1.15 \times 10^{1} \times 10^{9}$ **Write 11.5 as 1.15×10^{1}.**

$= 1.15 \times 10^{10}$ **Add the exponents.**

The Great Pyramid contains about 1.15×10^{10} lb of stone.

Real-World Connection

In ancient times, the Great Pyramid of Giza was plundered inside and out. Outside, most of the casing of smooth, white limestone was removed. The height of the pyramid is now about 30 ft less than the original height.

✓ Check Understanding **Example 7**

7. **Chemistry** A hydrogen atom has a mass of 1.67×10^{-27} kg. What is the mass of 6×10^{3} hydrogen atoms? Express the result in scientific notation.

EXERCISES

For more exercises, see *Extra Practice.*

Practice and Problem Solving

A Practice by Example

Examples 1 and 2 (page 216)

In Exercises 1–7, write each number in scientific notation.

1. 8,900,000,000 **2.** 555,900,000 **3.** 0.000631

4. 0.000006 **5.** 0.209 **6.** 0.00409

7. Solar System Pluto is about 5 billion km from the sun.

Example 3 (page 216)

Write each number in standard notation.

8. 5.94×10^7 **9.** 2.104×10^{-8} **10.** 1.2×10^5

11. 7.2×10^{-4} **12.** 2.75×10^8 **13.** 6.0502×10^{-3}

Example 4 (page 217)

Write each number in scientific notation.

14. 0.09×10^{12} **15.** 0.72×10^{-4}

16. 52.8×10^9 **17.** $3{,}508 \times 10^{-7}$

Example 5 (page 217)

Order from least to greatest.

18. $16 \times 10^9, 2.3 \times 10^{12}, 0.065 \times 10^{11}$

19. $253 \times 10^{-9}, 3.7 \times 10^{-8}, 12.9 \times 10^{-7}$

20. $65 \times 10^4, 432 \times 10^3, 2.996 \times 10^4$

Example 6 (page 218)

Multiply. Express each result in scientific notation.

21. $(5 \times 10^6)(6 \times 10^2)$ **22.** $(4.3 \times 10^3)(2 \times 10^{-8})$

23. $(9 \times 10^{-3})(7 \times 10^8)$ **24.** $(3 \times 10^2)(2 \times 10^2)$

Example 7 (page 218)

25. Zoology An ant weighs about 2×10^{-5} lb. There are about 10^{15} ants on Earth. How many pounds of ants are on Earth?

B Apply Your Skills

In Exercises 26–30, write each number in standard notation.

26. 9×10^2 **27.** 8.43×10^6 **28.** 6.02×10^{-7}

29. Astronomy One light year is 5.88×10^{12} mi.

30. Zoology The most venomous scorpion delivers 9×10^{-6} oz of venom per bite.

Order from least to greatest.

31. $10^9, 10^{-8}, 10^5, 10^{-6}, 10^0$

32. $55.8 \times 10^{-5}, 782 \times 10^{-8}, 9.1 \times 10^{-5}, 1{,}009 \times 10^2, 0.8 \times 10^{-4}$

33. Writing in Math Explain how to write each number in scientific notation.

a. 0.00043 **b.** 523.4×10^5

C Challenge **Solve. Write each result in scientific notation.**

34. **Statistics** The population density of India is about 8.33×10^2 people per square mile. The area of India is 1.2×10^6 mi^2. What is the approximate population of India?

35. **Health Care** In the year 2005, the population of the United States is expected to be about 296 million. Health expenditures will be about $7,350 per person. In total, about how much will the United States spend on health care in 2005?

Test Prep

Multiple Choice

36. What is 55,000 in scientific notation?
   A. 5.5×10^{-4} **B.** 55×10^3 **C.** 0.55×10^4 **D.** 5.5×10^4

37. What is 2×10^{-4} in standard notation?
   F. 8,000 **G.** 0.0008 **H.** 0.0002 **I.** 2,000

Reading Comprehension Read the passage before doing Exercises 38 and 39.

One Giant Leap

On July 20, 1969, Neil Armstrong and Edwin “Buzz” Aldrin, Jr., were first to set foot on the moon. With his first step, Armstrong announced over the radio, “That’s one small step for a man, one giant leap for mankind.”

The moon is about 380,000 km from Earth. The footsteps the astronauts left on the moon will probably be visible for at least 10 million years.

38. What is the distance in meters from Earth to the moon? Use scientific notation.

39. How many 0.5-meter footsteps would it take to walk from Earth to the moon? Use scientific notation.

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Mixed Review

Lesson 4-8 **Simplify each expression.**

40. $\frac{10^7}{10^9}$ **41.** $\frac{x^3y}{xy}$ **42.** $\frac{15b^2}{10b^5}$ **43.** $\frac{9m^7}{3m^5n}$

Lesson 3-4 **Algebra** **Use the formula $d = rt$. Find d, r, or t.**

44. $r = 46.2$ m/h, $t = 2.75$ h **45.** $d = 4.68$ ft, $t = 5.2$ h **46.** $d = 988$ cm, $r = 6.5$ cm/s

Lesson 1-8 **47.** **Patterns** A clock strikes a chime once at one o’clock, twice at two o’clock, and so on. In a twelve-hour period, what is the total number of chimes the clock strikes?

Technology

Scientific Notation With Calculators

For Use With Lesson 4-9

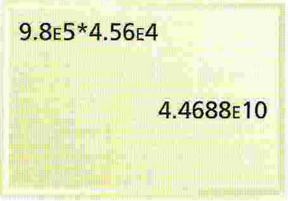
When you enter a number with more digits than a calculator can display, the calculator translates the number into scientific notation. "E11" in the output below means "$\times 10^{11}$."

112,345,678,999 ENTER → 1.12345679E11 **The display shows the number rounded.**

You can use a calculator to calculate with numbers in scientific notation.

1 EXAMPLE

Use a calculator to find $(9.8 \times 10^5)(4.56 \times 10^4)$.



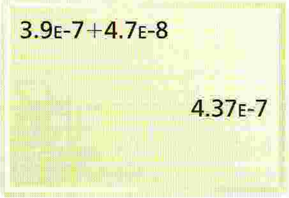
```
9.8E5*4.56E4
                4.4688E10
```

Use EE 5 to enter E5 and EE 4 to enter E4.

The product is 4.4688×10^{10}.

2 EXAMPLE

Use a calculator to find $3.9 \times 10^{-7} + 4.7 \times 10^{-8}$.



```
3.9E-7+4.7E-8
                  4.37E-7
```

Use (–) for negative exponents.

The sum is 4.37×10^{-7}.

EXERCISES

Use a calculator to simplify. Write each result in scientific notation.

1. $1.5 \times 10^{11} - 2.4 \times 10^8$

2. $6.97 \times 10^5 + 4.8 \times 10^{10}$

3. $(1.02 \times 10^9)(1.98 \times 10^7)$

4. $(5.1 \times 10^3) \div (3.64 \times 10^{10})$

5. $(2.8 \times 10^{13})(3.335 \times 10^{10})$

6. $9.807 \times 10^7 + 7.08 \times 10^{10}$

7. $7.1 \times 10^{-5} - 9.1 \times 10^{-6}$

8. $3.5 \times 10^{-6} + 6.76 \times 10^{-4}$

9. $(2.43 \times 10^{-3})(4.9 \times 10^{-10})$

10. $(1.08 \times 10^4) \div (7.3 \times 10^{-7})$

11. $(5.01 \times 10^{-3})(8.5 \times 10^{-8})$

12. $1.99 \times 10^{-5} - 3.81 \times 10^{-4}$

Test-Taking Strategies

Reading-Comprehension Questions

Reading-comprehension questions require that you read and understand information given to you in print in order to use mathematics to solve the problem.

EXAMPLE

Read the passage below. Then answer the questions based on what is stated or implied in the passage.

> **Solar System Masses** The sun is the largest object in our solar system. It contains approximately 98% of the total solar-system mass. The interior of the sun can hold over 1.3 million Earths. The mass of Earth is 5.98×10^{24} kg. The sun is approximately 330,000 times the mass of Earth.

What is the mass of the sun?

You read, "The sun is approximately 330,000 times the mass of Earth."

$$\begin{array}{rcrcl} \text{Mass of sun} & \approx & 330{,}000 & \times & \text{mass of Earth.} \\ M & \approx & 330{,}000 & \times & 5.98 \times 10^{24} \\ & \approx & 3.3 \times 10^{5} & \times & 5.98 \times 10^{24} \\ & \approx & 19.734 \times 10^{29} & & \\ & \approx & 1.97 \times 10^{30} & & \end{array}$$

The mass of the sun is about 1.97×10^{30} kg.

EXERCISES

Read the passage below. Then answer the questions based on what is stated or implied in the passage.

> **Planet Distances** The diameter of Jupiter is 142,800 km. Saturn is almost as big with a diameter of 120,000 km. Earth, by comparison, has a diameter of only 12,756 km. Jupiter's mass is 318 times the mass of Earth. Saturn's mass is only 95 times the mass of Earth.

1. Put the planets named in the above passage in order from smallest to largest, based on their diameters.
2. The mass of Jupiter is about how many times the mass of Saturn?
3. The diameter of Jupiter is about how many times the diameter of Saturn?
4. What could you conclude about Saturn from this passage?

Chapter 4

Chapter Review

Vocabulary

- **base** (p. 182)
- **composite number** (p. 186)
- **divisible** (p. 178)
- **equivalent fractions** (p. 192)
- **exponents** (p. 182)
- **factor** (p. 179)
- **greatest common factor (GCF)** (p. 187)
- **power** (p. 182)
- **prime factorization** (p. 187)
- **prime number** (p. 186)
- **rational number** (p. 201)
- **scientific notation** (p. 215)
- **simplest form** (p. 192)
- **standard notation** (p. 216)

Reading Math
Understanding Vocabulary

Choose the vocabulary term that correctly completes each sentence.

1. One integer is a _?_ of another integer if it divides that integer with remainder zero.

2. A fraction is in _?_ when the numerator and denominator have no factors in common other than 1.

3. A number that you can write as the quotient $\frac{a}{b}$ of two integers, where b is not zero, is a _?_.

4. You can write numbers using powers of 10 in a shorthand way called _?_.

5. You can show repeated multiplication with _?_.

6. If a positive integer greater than 1 has exactly two factors, 1 and the integer itself, the integer is a _?_.

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Skills and Concepts

4-1 Objectives

- To use divisibility tests (p. 178)
- To find factors (p. 179)

One integer is **divisible** by another if the remainder is zero when you divide. Divisibility tests help you find factors. One integer is a **factor** of another integer if it divides that integer with remainder zero.

List the positive factors of each number.

7. 12 **8.** 30 **9.** 42 **10.** 72 **11.** 111 **12.** 252

4-2 Objectives

- To use exponents (p. 182)
- To use the order of operations with exponents (p.183)

To simplify an expression that has an **exponent,** remember that the **base** is the number used as a factor. The exponent shows the number of times the base is used as a factor.

Simplify each expression.

13. 2^3

14. $3(10 - 7)^2$

15. $28 + (1 + 5)^2 \cdot 4$

16. -5^2

Evaluate each expression.

17. x^2, for $x = 11$

18. $7m^2 - 5$, for $m = 3$

19. $(2a + 1)^2$, for $a = -4$

20. b^2, for $b = -4$

4-3 Objectives

- ▼ To find the prime factorization of a number (p. 186)
- ▼ To find the greatest common factor (GCF) of two or more numbers (p. 187)

A **prime number** is an integer greater than 1 with exactly two positive factors, 1 and itself. An integer greater than 1 with more than two factors is a **composite number.** The **prime factorization** of a composite number is the product of its prime factors.

The **greatest common factor (GCF)** of two or more numbers or expressions is the greatest factor that the numbers or expressions have in common. You can list factors or use prime factorization to find the GCF of two or more numbers or expressions.

Is each number *prime, composite,* or *neither?* For each composite number, write the prime factorization. Use exponents where possible.

21. 13

22. 20

23. 73

24. 110

25. 87

Find the GCF.

26. 16, 60

27. 36, 81, 27

28. 15, 17, 30

29. $3x^2y$, $9x^2$

30. $8a^2b$, $14ab^2$

31. $3cd^4, 12c^3d, 6c^2d^2$

32. **Reasoning** Why is the GCF of two or more positive integers never greater than the least of the numbers?

4-4 Objectives

- ▼ To find equivalent fractions (p. 192)
- ▼ To write fractions in simplest form (p. 192)

Equivalent fractions describe the same part of a whole. A fraction is in **simplest form** when the numerator and the denominator have no common factors other than 1. You can use the GCF of the numerator and denominator to write a fraction in simplest form.

Write in simplest form.

33. $\frac{3}{15}$

34. $\frac{10}{20}$

35. $\frac{16}{52}$

36. $\frac{28}{40}$

37. $\frac{21}{33}$

38. $\frac{9}{54}$

39. $\frac{xy}{y}$

40. $\frac{25m}{5m}$

41. $\frac{2y}{8y}$

42. $\frac{2c}{5c}$

43. $\frac{9x^2}{27x}$

44. $\frac{36bc}{9c}$

4-5 Objectives

- ▼ To find all possibilities when you solve a problem (p. 197)

To account for all possibilities in a word problem, make an organized list or a diagram to keep track of possibilities as you find them.

45. **School** Mike, Don, Tameka, and Rosa sit in the four desks in the last row of desks. Each day they sit in a different order. How many days can they do this before they repeat a seating pattern?

4-6 Objectives

- To identify and graph rational numbers (p. 201)
- To evaluate fractions containing variables (p. 202)

A **rational number** is any number you can write as a quotient $\frac{a}{b}$ of two integers, where b is not zero.

Graph the rational numbers below on the same number line.

46. 2 **47.** -0.6 **48.** $-\frac{5}{10}$ **49.** $\frac{2}{10}$

Evaluate each expression for $a = -5$ and $b = -2$. Write in simplest form.

50. $\frac{b}{a}$ **51.** $\frac{a+b}{4b}$ **52.** $\frac{b-a}{a-b}$ **53.** $\frac{b^2}{a}$

4-7 and 4-8 Objectives

- To multiply powers with the same base (p. 205)
- To find a power of a power (p. 206)
- To divide expressions containing exponents (p. 210)
- To simplify expressions with integer exponents (p. 211)

To multiply numbers or variables with the same base, add the exponents. To raise a power to a power, multiply the exponents. To divide numbers or variables with the same nonzero base, subtract the exponents.

Simplify each expression.

54. $2^4 \cdot 2^3$ **55.** $7a^4 \cdot 3a^2$ **56.** $b \cdot c^2 \cdot b^6 \cdot c^2$ **57.** $(x^3)^5$

58. $(y^4)^5$ **59.** $\frac{4^8}{4^2}$ **60.** $\frac{b^2}{b^4}$ **61.** $\frac{28xy^7}{32xy^{12}}$

4-9 Objectives

- To write and evaluate numbers in scientific notation (p. 215)
- To calculate with scientific notation (p. 218)

Scientific notation provides a way to write numbers as the product of two factors, a power of 10 and a decimal greater than or equal to 1, but less than 10. To multiply numbers in scientific notation, multiply the decimals, multiply the powers of ten, and then put the result into scientific notation.

Write each number in scientific notation.

62. 2,000,000 **63.** 458,000,000 **64.** 0.0000007 **65.** 0.0000000059

Write each number in standard notation.

66. 8×10^{11} **67.** 3.2×10^{-6} **68.** 1.119×10^7 **69.** 5×10^{-12}

Order from least to greatest.

70. $3{,}644 \times 10^9, 12 \times 10^{11}, 4.3 \times 10^{10}$

71. $58 \times 10^{-10}, 8 \times 10^{-10}, 716 \times 10^{-10}$

Multiply. Express each result in scientific notation.

72. $(4 \times 10^9)(6 \times 10^6)$ **73.** $(5 \times 10^7)(3.6 \times 10^3)$

Chapter 4

Chapter Test

Take It to the NET
Online chapter test at
www.PHSchool.com
Web Code: ada-0452

State whether each number is divisible by 2, 3, 5, 9, or 10.

1. 36
2. 100
3. 270
4. 84
5. 555
6. 49

List all the factors of each number.

7. 16
8. 30
9. 41
10. 23
11. 55
12. 64

Simplify each expression.

13. 5^3
14. $2^0 \cdot 2^3$
15. $3^2 + 3^3$
16. $4^2 \cdot 1^3$
17. $(-9)^2$
18. $(7 - 6)^4$
19. $-2(3 + 2)^2$
20. $-6^2 + 6$

21. **Writing in Math** A number written in scientific notation is doubled. Must the exponent of the power of 10 change? Explain.

Evaluate for $a = -2$ and $b = 3$.

22. $(a \cdot b)^2$
23. a^2b
24. $b^3 \cdot b^0$
25. $(a + b)^5$
26. $b^2 - a$
27. $2(a^2 + b^3)$

Is each number *prime* or *composite*? For each composite number, write the prime factorization.

28. 24
29. 17
30. 42
31. 54
32. 72
33. 100

Find each GCF.

34. 56, 96
35. 36, 60
36. 14, 25
37. $15x, 24x^2$
38. $14a^2b^3, 21ab^2$

Simplify.

39. $\frac{4}{16}$
40. $\frac{44}{52}$
41. $\frac{15}{63}$
42. $\frac{a^3}{a^2}$
43. $\frac{5b^4}{b}$
44. $\frac{8m^4n^2}{40mn}$

Graph the numbers on the same number line.

45. $\frac{1}{10}$
46. -0.3
47. $-\frac{1}{2}$
48. 1

49. A car manufacturer offers exterior colors of white, blue, red, black, and silver. The manufacturer offers interior colors of black and silver. How many different color combinations are there?

Evaluate for $x = 4$ and $y = -3$. Write in simplest form.

50. $\frac{2y}{x^2}$
51. $\frac{xy}{5x}$
52. $\frac{(x + y)^3}{x}$
53. $\frac{x + 3y}{10}$
54. $\frac{y^2 - x}{5}$
55. $\frac{x - y}{x + y}$

Simplify each expression.

56. $a^4 \cdot a$
57. $(y^3)^6$
58. $x^3 \cdot x^6 \cdot y^2$
59. $(a^3)^2$
60. $6b^7 \cdot 5b^2$
61. $\frac{9^8}{9^2}$
62. $\frac{6a^7}{15a^3}$
63. $\frac{b^8}{b^{11}}$
64. $\frac{2x^2y^5}{8x^3y^5}$

Write each number in scientific notation.

65. 43,000,000
66. 6,000,000,000
67. 0.0000032
68. 0.00000000099

Write each number in standard notation.

69. 5×10^5
70. 3.812×10^{-7}
71. 9.3×10^8
72. 1.02×10^{-9}

Order from least to greatest.

73. $3 \times 10^{10}, 742 \times 10^7, 0.006 \times 10^{12}$
74. $85 \times 10^{-7}, 2 \times 10^{-5}, 0.9 \times 10^{-8}$

Multiply. Express each result in scientific notation.

75. $(3 \times 10^{10})(7 \times 10^8)$
76. $(8.3 \times 10^6)(3 \times 10^5)$

Test Prep

CUMULATIVE REVIEW

CHAPTERS 1–4

Multiple Choice

Choose the best answer.

1. Which expression is equivalent to $\frac{x^3y^7}{x^5y^2}$?

A. $x^{-2}y^5$ **B.** x^2y^5

C. $x^{-2}y^{-5}$ **D.** x^2y^{-5}

2. What is the simplest form of $(4c - 5c) + (7 - 2)$?

F. $c + 5$ **G.** $-c + 5$

H. $9c + 5$ **I.** $c - 5$

3. Which integer is *not* a solution of $25 + t < 19$?

A. -43 **B.** -7

C. -8 **D.** -6

4. Which sentence is true?

F. $16 \geq 2 \cdot 9$

G. $-36 - 10 = 4(5)$

H. $5[-6 - (-2)] = 2 \cdot (-5)2$

I. $32 - (-4 \cdot 6) \leq 54$

5. Which number is divisible by both 3 and 9?

A. 24,000 **B.** 36,089

C. 45,288 **D.** 95,500

6. Which expression is equivalent to $-8 \cdot n \cdot n \cdot n \cdot 4 \cdot t$?

F. $-32n^3t$ **G.** $-8n^3 + 4t$

H. $-32 \cdot 3n \cdot t$ **I.** $-32nt^3$

7. Which expression is the GCF of $24x^3$ and $64x$?

A. $1{,}536x^4$ **B.** $4x^4$

C. $40x^2$ **D.** $8x$

8. Which expression is equivalent to x^{12}?

F. $x^6 + x^6$ **G.** $(x^4)^8$

H. $x^2 \cdot x^6$ **I.** $x^6 \cdot x^6$

9. Which symbol makes $7^2 \cdot 7^5$ ■ $(7^5)^2$ true?

A. $>$ **B.** $<$

C. $=$ **D.** $\geq$

10. What is the simplest form of $x^5 \cdot y \cdot x^5 \cdot y$?

F. $(x^{25})(2y)$ **G.** x^5y^2

H. $2x^5y$ **I.** $x^{10}y^2$

11. What is the simplest form of $\frac{w^{12}y^{15}z}{w^9y^7}$?

A. w^3y^8z **B.** $w^{21}y^{22}z$

C. $\frac{w^{21}y^{22}z}{wz}$ **D.** $\frac{w^3y^8z}{wz}$

12. What is the prime factorization of 90?

F. $2 \cdot 3^2 \cdot 5$ **G.** $2 \cdot 5 \cdot 9$

H. $3 \cdot 3 \cdot 5^2$ **I.** $2 \cdot 45$

13. Which number is divisible by 2, 3, and 5?

A. 70 **B.** 105 **C.** 120 **D.** 235

Gridded Response

14. Simplify 2^{-3}.

15. Evaluate $\frac{3m - 12}{n}$, for $m = 8$ and $n = 4$.

16. Simplify $2(11 + 7 \cdot 2)$.

17. Evaluate $-a^2 + 4$ for $a = -1$.

18. Simplify $8 + (-8) - (-8)$.

Short Response

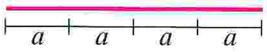
19. **a.** The product of -6.2 and a number k is -70.68. Write an equation to find k.

b. Solve for k.

20. What is the GCF of 45 and 54? Explain.

21. **a.** Write a variable expression for the length of the red segment.

b. What is the segment length if $a = 7$?



Extended Response

22. The store sells erasers for \$.05, \$.10, and \$.15. In how many ways can you spend \$.20 to buy erasers? Explain your answer using a table.

Real-World Snapshots

Comparing Life Cycles

Applying Factors and Multiples All living things go through stages of growth and development marked by changes in how they look. Few animals, however, have a life cycle as unusual as the periodic cicadas (sih-CAY-duhs) of North America.

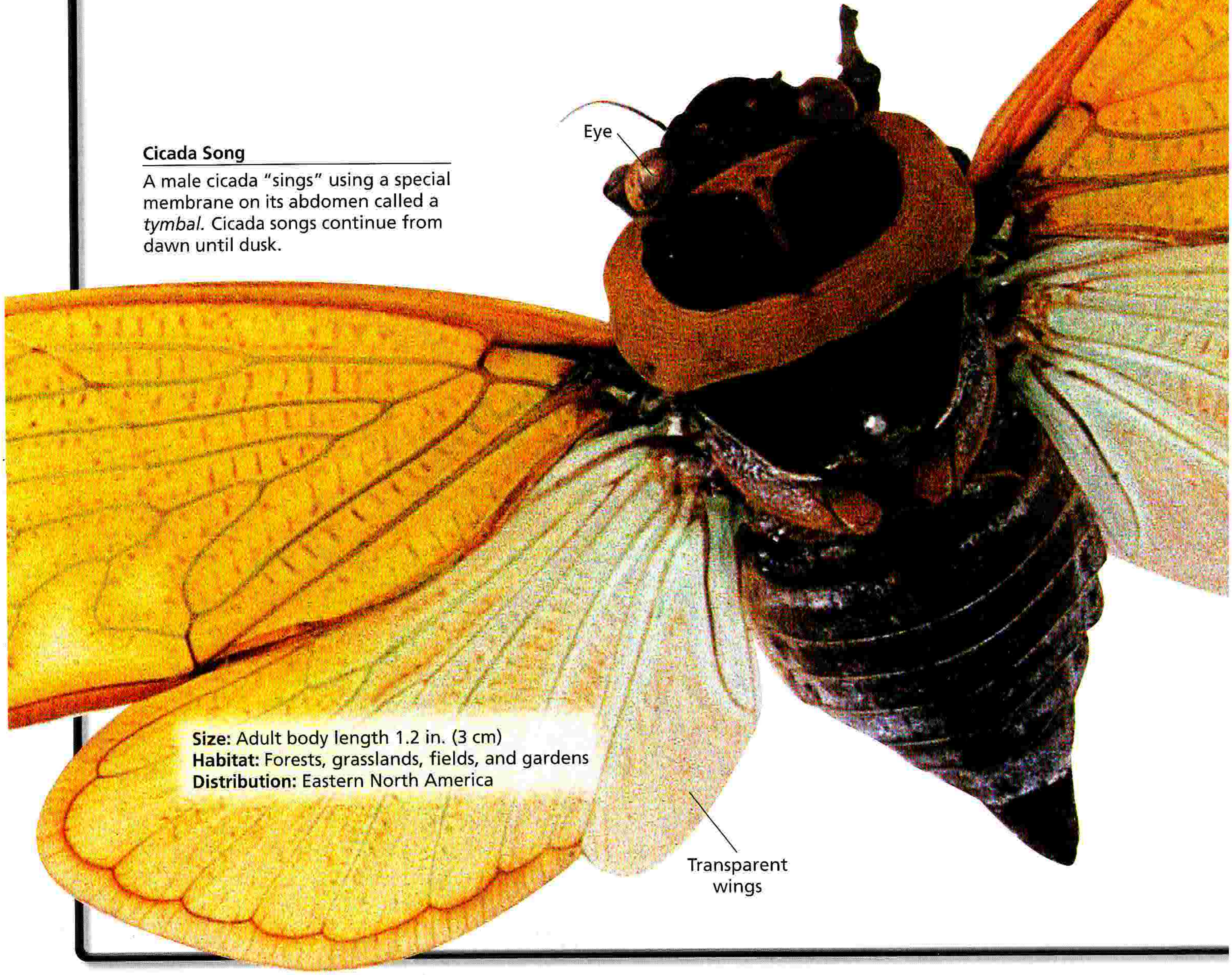
Some of these insects spend 13 or 17 years living underground and feeding on roots. Then, within a matter of days, thousands of them emerge above ground. These large groups of periodic cicadas are called broods. They molt into winged adults but live for only a few more weeks.

Periodic Cicada

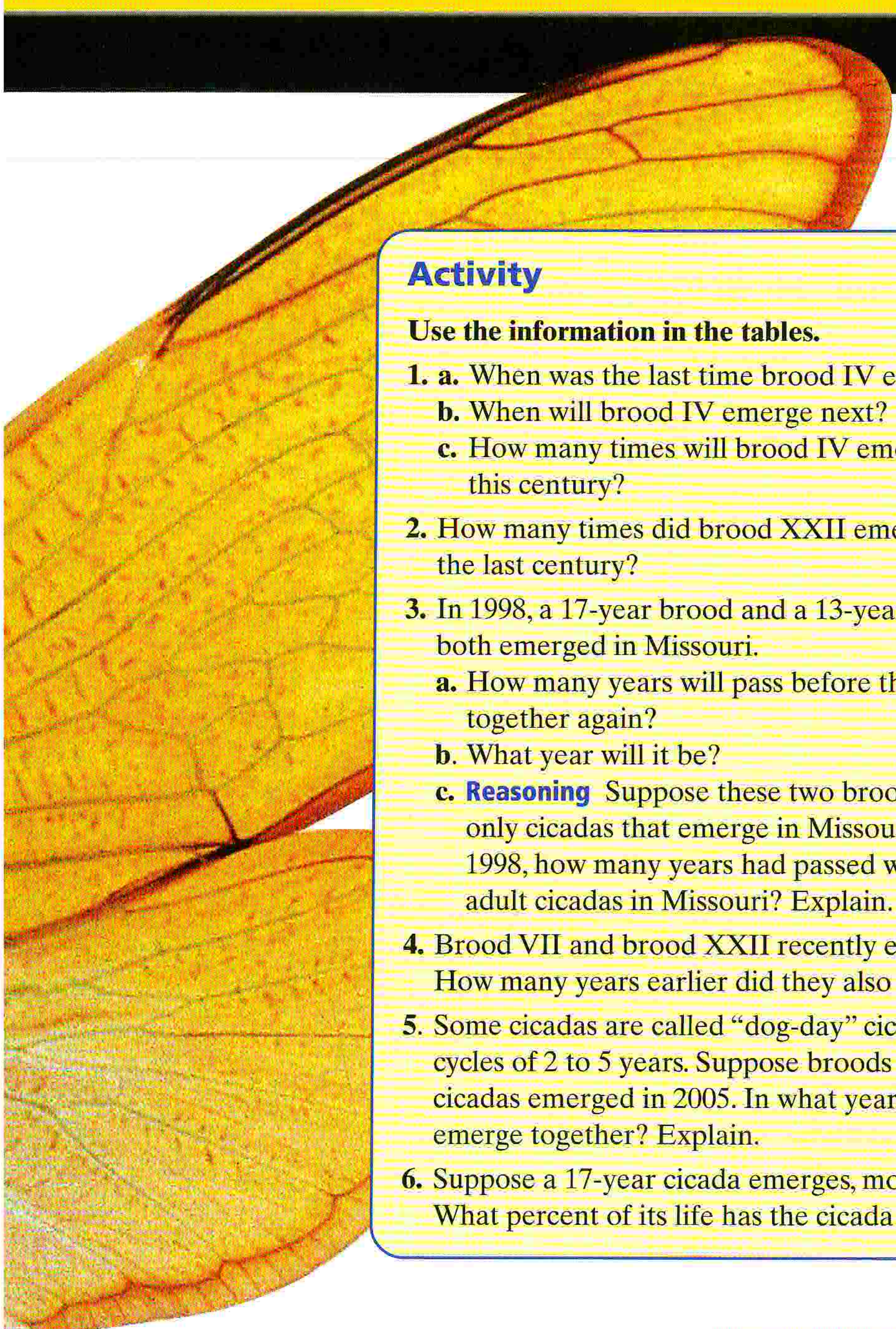
An adult cicada perches on a branch to dry its wings.

Cicada Song

A male cicada "sings" using a special membrane on its abdomen called a *tymbal.* Cicada songs continue from dawn until dusk.



Size: Adult body length 1.2 in. (3 cm)
Habitat: Forests, grasslands, fields, and gardens
Distribution: Eastern North America



Activity

Use the information in the tables.

1. **a.** When was the last time brood IV emerged?
  b. When will brood IV emerge next?
  c. How many times will brood IV emerge during this century?

2. How many times did brood XXII emerge during the last century?

3. In 1998, a 17-year brood and a 13-year brood both emerged in Missouri.
  a. How many years will pass before they emerge together again?
  b. What year will it be?
  c. **Reasoning** Suppose these two broods are the only cicadas that emerge in Missouri. Before 1998, how many years had passed without adult cicadas in Missouri? Explain.

4. Brood VII and brood XXII recently emerged in the same year. How many years earlier did they also emerge together?

5. Some cicadas are called "dog-day" cicadas. These cicadas have cycles of 2 to 5 years. Suppose broods of 3-year, 4-year, and 5-year cicadas emerged in 2005. In what year would all three broods next emerge together? Explain.

6. Suppose a 17-year cicada emerges, molts, and dies after 4 weeks. What percent of its life has the cicada spent as an adult?

17-Year Cicadas

Brood	Year Seen
III	1997
IV	1998
VII	2001

13-Year Cicadas

Brood	Year Seen
XIX	1998
XXII	2001
XXIII	2002

Gathering Insects

Students taking a class on entomology collect and study insects.

Take It to the NET For more information about cicadas, go to **www.PHSchool.com.**
Web Code: ade-0453

Where You've Been

- In Chapter 1, you learned how to add, subtract, multiply, and divide integers.
- In Chapter 2, you solved equations by adding, subtracting, multiplying, and dividing.
- In Chapter 4, you investigated exponents.

Instant self-check online and on CD-ROM

Diagnosing Readiness

(For help, go to the lesson in green.)

Solving Equations (Lessons 3-5 and 3-6)

Solve each equation.

1. $x + 1.8 = 3$ **2.** $n - 41 = 19$ **3.** $a \div (-3) = 15$ **4.** $-19 = p + 21$

5. $6t = 9$ **6.** $40 = z - 34$ **7.** $8d = 64$ **8.** $-0.89 = \frac{x}{2}$

Finding the Greatest Common Factor (Lesson 4-3)

Find the GCF of each group of numbers.

9. 3, 15 **10.** 16, 20 **11.** 12, 36 **12.** 11, 30 **13.** 30, 40, 210

14. 45, 80 **15.** 27, 72 **16.** 15, 121 **17.** 30, 500 **18.** 14, 28, 84

Reading and Writing Fractions (Lesson 4-4)

Write two equivalent fractions to describe each model.

19.

20.

21.

Writing Fractions and Decimals (Lesson 4-6)

Write each fraction in simplest form.

22. $\frac{10}{12}$ **23.** $\frac{8}{20}$ **24.** $-\frac{32}{16}$ **25.** $\frac{25}{100}$ **26.** $-\frac{120}{125}$ **27.** $\frac{15}{45}$

28. $\frac{-20}{-75}$ **29.** $\frac{16}{124}$ **30.** $-\frac{18}{81}$ **31.** $-\frac{10}{65}$ **32.** $\frac{14}{84}$ **33.** $\frac{55}{77}$

Divide. Write each quotient as a decimal.

34. $27 \div 5$ **35.** $6 \div 10$ **36.** $10 \div 16$ **37.** $9 \div 12$ **38.** $15 \div 40$